Digital Baseband/IF Modem: ADC and Uniform Sampler

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- Review. Modulation, Demodulation, and PSD
  - Complex envelope, quadrature modulator, polar modulator
  - PAPR, PMEPR, phase continuity
  - PSD of ASK, PSK, PAM, QAM, OQPSK, MSK, DS-CDMA, and OFDM
  - homodyne receiver, heterodyne receiver
  - real-valued bandpass WSS random process and its complex envelope
  - AWGN, proper-complex AWGN

- Preview.
  - Digital BB/IF Modem, ADC, DAC
  - Uniform sampling and ADC
  - Sampling viewed in FD
  - The Nyquist sampling theorem

- Modem
  - A **modem** consists of a **mod**ulator part and a **dem**odulator part to enable two-way communications.
  - The modulator part is connected to an information source and a Tx antenna.
  - The demodulator part is connect to an Rx antenna and an information sink.
  - The Tx and the Rx antenna are not necessarily different.
  - A modem consists of a **digital** system part and an **analog** system part.
    * The two parts are separated into **two chips in two packages** or put in a **single package**, or combined in **one chip**.
    * The analog part is connect to the antenna(s).
    * The two parts are linked by digital-to-analog converter(s) **DAC**(s) and analog-to-digital converter(s) **ADC**(s).

- Digital Baseband Modem vs. Digital IF Modem
  - If two DACs generate two CT baseband signals (I and Q) and two ADCs admit two CT baseband signals (I and Q),
  - then the modem is called a **digital baseband modem**.

  - If one DAC generates a CT IF signal and one ADC admits a CT IF signal,
  - then the modem is called a **digital IF modem**.

  - A hybrid can be possible.
  - Multiple Tx and/or antennas may be connected to the analog part.

- Uniform Sampler and ADC
  - Def. A **uniform sampler with**
    * **sampling offset** Δ and **sampling rate** $1/T_s$

    is a linear system with
    * a **CT input signal** $x(t)$ and a **DT output signal** $x_d[n]$,

    where the I/O relation is given by

$$x_d[n] = x(nT_s + \Delta).$$

  - Remarks
    * Different offsets generate different DT signals.
    * Different rates generate different DT signals.

* Uniform sampling is often called sampling.
  - However, keep in mind that there exist many **non-uniform sampling** schemes.

* The signals are real-valued or complex-valued, deterministic or random signals.
  - We prove theorems, such as **the Nyquist sampling theorem**, only for deterministic signals.
  - For random signals, a similar theorem exists, named **the Shannon-Nyquist sampling theorem**.

* An **ADC** is a real-world approximation of a uniform sampler with a real-valued input and a real-valued output.
  - A real-world ADC can be viewed as **a uniform sampler followed by a scalar quantizer**.
  - Thus, each sample consists of a finite number of bits.

- Sampling viewed in FD
  - Q. Find the DTFT $X_d(e^{\mathrm{j}2\pi f})$ of the output in terms of the sampling rate $1/T_s$ and the CTFT $X(f)$ of the input.
  - A. For simplicity, we set $\Delta = 0$. Then,

$$
\begin{aligned}
X_d(e^{\mathrm{j}2\pi f}) &= \sum_{n=-\infty}^{\infty} x_d[n] e^{-\mathrm{j}2\pi f n} = \sum_{n=-\infty}^{\infty} x(nT) e^{-\mathrm{j}2\pi f n} \\
&= \sum_{n=-\infty}^{\infty} \left(\int_{-\infty}^{\infty} X(\widetilde{f}) e^{\mathrm{j}2\pi \widetilde{f} n T_s} \, \mathrm{d}\widetilde{f} \right) e^{-\mathrm{j}2\pi f n} \\
&= \frac{1}{T_s} \int_{-\infty}^{\infty} X\left(\frac{\widehat{f}}{T_s}\right) \sum_{n=-\infty}^{\infty} e^{\mathrm{j}2\pi(\widehat{f}-f)n} \, \mathrm{d}\widehat{f} \qquad \left(\widetilde{f} = \frac{\widehat{f}}{T}\right) \\
&= \frac{1}{T_s} \int_{-\infty}^{\infty} X\left(\frac{\widehat{f}}{T_s}\right) \sum_{m=-\infty}^{\infty} \delta(\widehat{f} - f - m) \, \mathrm{d}\widehat{f} \\
&= \ldots \\
&= \frac{1}{T_s} \sum_{m=-\infty}^{\infty} X\left(\frac{f-m}{T_s}\right).
\end{aligned}
$$

– Remarks

* The DTFT of the output factors into the **sampling rate** $1/T_s$ and the **scaled folded spectrum** $\sum_{m=-\infty}^{\infty} X((f-m)/T_s)$ of the input.

* The scaled folded spectrum can be obtained
  · by first periodically extending the CTFT of the input to have the period equal to the sampling rate, i.e., $\sum_{m=-\infty}^{\infty} X(f-m/T_s)$,
  · and then by scaling the horizontal axis to make its period unity.

* Alternatively, the scaled folded spectrum can be obtained
  · by first scaling the horizontal axis of the CTFT of the input to make the sampling rate unity, i.e., $X(f/T_s)$,
  · and then by periodically extending it to have the period equal to unity.

– **Nyquist's Sampling Theorem**.

Suppose that a CT signal $x(t)$ is **finite-energy and bandlimited to** $f \in [-W, W]$, i.e.,

$$E_{xx} = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-W}^{W} |X(f)|^2 df.$$

Then, **the uniform sampling** $x_d[n] \triangleq x(nT_s + \Delta)$ **preserves all the information on** $x(t)$ **iff**

$$\frac{1}{T_s} \text{ [samples/second]} \geq 2W \text{ [hertz]}$$

i.e., the sampling rate, whose unit often denoted by **[S/s]**, is greater than or equal to twice the bandwidth in [Hz].

– A **proof by construction** can be found in the next lecture.

∗ Obviously, the inequality holds iff no **aliasing** occurs.

– Remarks

* This theorem holds for any CT signal satisfying the two conditions.
  - $2W$ is called **Nyquist's minimum sampling rate for zero aliasing or for perfect reconstruction**.
  - An over-estimated bandwidth may allow a smaller sampling rate.

* An additional condition or structure in the signal may reduce the sampling rate with or without a proper pre-processing.
  - If $x(t)$ is real-valued baseband signal, then the sampling rate $2W$ **real samples per second** can be apparently reduced by first converting $x(t)$ to its pre-envelope $x_+(t)$ and then sampling at W **complex samples per second**. Of course, no rate reduction is achieved effectively because one complex sample equals two real samples.

- If $x(t)$ is real-valued bandpass signal with bandwidth $W' < W$, then the sampling rate $2W$ **real samples per second** can be reduced by first converting $x(t)$ to its complex-envelope $x_l(t)$ and then sampling at W' complex samples per second, equivalently, at $2W'$ **real samples per second**.

- There are other additional conditions that can reduce the sampling rate.

* For band-unlimited signals, an **anti-aliasing filter** precedes a uniform sampler.