Digital Baseband/IF Modem: Motivations for Upsampling and Downsampling

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- Review.
  - Modulation, Demodulation, and PSD
  - Digital Modem
  - ADC and uniform sampling
  - DAC and uniform interpolation
  - The Nyquist sampling theorem

- Preview.
  - Tx Part of Digital BB Modem
  - Tx Part of Digital IF Modem
  - Rx Part of Digital BB/IF Modem

- Tx Part of Digital BB Modem
  - Q. Suppose that the CT real BB signals to be generated by the Tx part of the modem are given by $x_c(t)$ and $x_s(t)$ such that

$$x_l(t) = x_c(t) + \mathrm{j}x_s(t),$$

    and that the modem can have two synchronous DACs.

    Choose the conversion rate of the DACs and also **find two DT signals** fed to the DACs.

  - A. Let $1/T_I$ be the conversion rate and $B = W/2$ be the bandwidth at complex baseband. Then, from Nyquist's sampling theorem, we can choose **any** $1/T_I$ **satisfying**

$$\frac{1}{T_I} \geq 2B = W$$

    and then the DT signals are given by

$$x_{c,d}[n] \triangleq x_c(nT_I) \text{ and } x_{s,d}[n] \triangleq x_s(nT_I), \forall n.$$

- Q. Is it really so?
- A. No, it isn't.
  - Actually, we cannot choose arbitrary $1/T_I$ satisfying the Nyquist condition because available DACs have a finite set of conversion rates.
  - Moreover, depending on the choice of $1/T_I$, the computational complexity in generating $x_{c,d}[n]$ and $x_{s,d}[n]$ can drastically vary.
    * In QAM $x_l(t) = \sum_{n=-\infty}^{\infty} d[n]p(t - nT)$, it is often preferred to have
    $$T_I = \frac{T}{L} \text{ for some } L \in \mathbb{N},$$
    i.e., the conversion rate is an integer multiple of the symbol rate.
    * Then, $x_{l,d}[n] \triangleq x_{c,d}[n] + \mathrm{j}x_{s,d}[n]$ can be obtained by
    $$x_{l,d}[n] = x_l(nT/L) = ... = \sum_{m=-\infty}^{\infty} d[m]p_d[n - mL] \qquad (1)$$
    where $p_d[n] = p(nT/L)$.

- L is often misleadingly called **the oversampling factor**.
- Consequently, $p_d[n] = p(nT/L)$ is called the **L-times oversampled version** of $p(t)$.
- In most of the cases, B and $1/T$ satisfy

$$\frac{1}{2T} < B \leq \frac{2}{2T} \Longleftrightarrow 1 < 2BT = WT \leq 2. \tag{2}$$

Thus, L must be greater than 1 because

$$\frac{1}{T_I} = \frac{L}{T} \geq 2B > \frac{1}{T}.$$

- When $L = 1$ is chosen, usually $p(t) = \mathrm{sinc}(t/T)$ satisfying $2BT = WT = 1$.

- Let β satisfy $B = \dfrac{1+\beta}{2T}$. $\beta \triangleq 2BT - 1$ is called the **excess bandwidth**.

- For example, 22% excess bandwidth means $\beta = 0.22$, where the system spends 22% more bandwidth than the system with $\operatorname{sinc}(t/T)$ having bandwidth $1/(2T)$.

- Eq. (2) $\Longleftrightarrow 1 < 2BT = WT \leq 2 \Longleftrightarrow 0 < \beta \leq 1$.

- Tx Part of Digital IF Modem
  - Q. Suppose that the CT real bandpass signal to be generated by the Tx part of the IF modem is given by $\mathrm{Re}\{x_l(t)e^{\mathrm{j}2\pi f_{\mathrm{IF}}t}\}$ with bandwidth in passband $W = 2B$

    and that the modem has one DAC.

    Choose the conversion rate $1/T_I$ of the DAC and **the IF frequency** f_{IF}, and also **find a DT signal** fed to the DAC.

  - A. From Nyquist's sampling theorem, we can choose **any** $1/T_I$ **and f_{IF} satisfying**

$$f_{\mathrm{IF}} \geq B = \frac{W}{2} \text{ and } \frac{1}{T_I} \geq 2(f_{\mathrm{IF}} + B)$$

    and then the DT signal is given by

$$x_{\mathrm{IF},d}[n] \triangleq \mathrm{Re}\{x_l(nT_I)e^{\mathrm{j}2\pi f_{\mathrm{IF}}nT_I}\}, \forall n. \tag{3}$$

- Q. Is it really so?
- A. No, it isn't.
  - Actually, we cannot choose arbitrary $1/T_I$ satisfying the Nyquist condition because available DACs have a finite set of conversion rates.
  - Moreover, depending on the choice of $1/T_I$, the computational complexity in generating $x_{\mathrm{IF},d}[n]$ can drastically vary.
    * In QAM $x_l(t) = \sum_{n=-\infty}^{\infty} d[n]p(t - nT)$, it is often preferred to have
    $$T_I = \frac{T}{L} \text{ for some } L \in \mathbb{N}$$
    to reduce the complexity in generating $x_{l,d}[n] = x_l(nT_I)$.
    * Moreover, it may be preferred for $1/T_I$ and f_{IF} to satisfy
    $$f_{\mathrm{IF}} T_I \in \mathbb{Q},$$
    because only a finite number of samples from $e^{\mathrm{j}2\pi f_{\mathrm{IF}} t}$ suffices for the mixing
    $$x_{l,d}[n] e^{\mathrm{j}2\pi f_{\mathrm{IF}} n T_I}.$$
    in Eq. (3).

- Motivations
  - The following topics are further studied in **multi-rate signal processing**.
  - Interpolation and Upsampling
    * The summation in Eq. (1) for the Tx part of the digital BB modem looks similar to the DT-to-CT uniform interpolation we learned already.
      · It may be called **DT-to-DT uniform interpolation** of $d[n]$ with interpolation function $p_d[n]$ and rate $1/L$.

    * In either in Eq. (1) or (3), $x_{l,d}[n]$ becomes too redundant if $1/T_I$ is too high compared with W.
      · In the Tx part of the digital IF modem, we may want to generate a low-rate sampled version of $x_l(t)$ first and then to use a **special-purpose digital system** that converts it to a DT signal that is equivalent to a high-rate sampled version.

– **D**ecimation and **D**ownsampling

* The Rx part of the digital BB modem may first oversample received CT signals to acquire timing and to estimate the channel, and then may want to reduce the sampling rate.
  · **We do not want to resample the CT signals** but to process the DT signals already acquired by oversampling.
  · This is an opposite to what happens in the Tx part of the digital BB modem.

* The Rx part of the digital IF modem samples a received and IF down-converted CT signal, digital downconverts it to BB, acquire timing, estimate the channel, and then reduce the sampling rate at complex baseband.
  · **We want to process a DT complex BB signal** to have an effectively low-rate sampled DT complex BB signal.
  · This is an opposite to what happens in the Tx part of the digital IF modem.