Digital Baseband/IF Modem: Upsampling and Interpolation

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- Review.
  - Modulation, Demodulation, and PSD
  - Digital BB/IF Modem
  - ADC, DAC, and the Nyquist sampling theorem
  - Tx/Rx Part of BB/IF Modem

- Preview.
  - Upsampling and (DT-to-DT) Interpolation viewed in TD and FD
  - Tx Part of Digital BB Modem
  - Tx Part of Digital IF Modem

- Upsampling
  - Def. **Upsampling by a factor of** L is a DT-to-DT linear operation having
    * a **DT input signal** $x_d[m]$ and
    * a **DT output signal** $y_d[n]$,

    where the I/O relation is given by

$$y_d[n+\Delta] \triangleq \begin{cases} x_d[n/L], & \text{for } n = mL, \\ 0, & \text{elsewhere,} \end{cases}$$

    with an integer sampling offset Δ or, equivalently,

$$y_d[n] = 0, \text{ for all } n \text{ except } y_d[mL+\Delta] = x_d[m], \forall m.$$

  - Fig. $\uparrow L$

- Remarks
  - Different rates generate different output signals.
  - Different offsets generate different output signals. Often, Δ is set to zero.
  - A similar operation to upsampling is DT-to-DT interpolation.
    - The FD view of upsampling will be covered after viewing the interpolation in the FD.

- Interpolation
  - Def. **Interpolation by a factor of** L is a DT-to-DT linear operation having
    - an **upsampler by a factor of** L and
    - a following **DT interpolation filter with impulse** $r[n]$.

    - Fig.

* That is, the I/O relation is given by

$$y_d[n] = \sum_{m=-\infty}^{\infty} x_d[m] r[n - mL],$$

where $r[n]$ is the **interpolation pulse/waveform/function** and $1/L$ is the **interpolation/conversion rate**.

– Remarks

* Upsampling by a factor of L with offset Δ corresponds to the interpolation with rate $1/L$ and function $\delta[n - \Delta]$.

* Often, upsampling means interpolation. In this lecture, the term upsampling is used as a narrow sense.

* Often, the interpolation by a factor of L
means the one having $r[n]$ the (ideal) LPF with height 1 and cutoff frequency $1/(2L)$.

* It is also convenient to consider $r[n] = \mathrm{sinc}(n/L)$,
which is the ideal LPF with height L and cutoff frequency $1/(2L)$.
In this lecture, the term interpolation is used as a wide sense.

– Q. Find the DTFT of $y_d[n]$ in terms of the upsampling factor L and the DTFT of $x_d[m]$.

– A. Similar to the DT-to-CT interpolation, we have

$$Y_d(e^{\mathrm{j}2\pi f}) = X_d(e^{\mathrm{j}2\pi fL})R(e^{\mathrm{j}2\pi f}). \tag{1}$$

The proof is omitted.

– Remarks

* $X_d(e^{\mathrm{j}2\pi fL})$ has period the same as the interpolation rate $1/L$.

* $r[n] = \mathrm{sinc}(n/L)$ iff $R(e^{\mathrm{j}2\pi f}) = L$ for $|f| \leq 1/(2L) \leq 1/2$.

* Interpolation with $\text{sinc}(n/L)$ and rate $1/L$ can be used for **sampling rate increase by** L.
  - Suppose that a CT signal $x(t)$ is given.
  - Let $x_d[m]$ be obtained by sampling $x(t)$ at rate $1/T_s$, i.e., $x_d[m] = x(mT_s) \Longleftrightarrow$

$$X_d(e^{\mathrm{j}2\pi f}) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} X\left(\frac{f-m}{T_s}\right).$$

  - Assume that there occurs **no aliasing in sampling at** $1/T_s$.

  - Let $z_d[n]$ be obtained by sampling $x(t)$ L times faster at rate L/T_s, i.e., $y_d[n] = x(nT_s/L) \Longleftrightarrow$

$$Z_d(e^{\mathrm{j}2\pi f}) = \frac{L}{T_s} \sum_{m=-\infty}^{\infty} X\left(\frac{(f-m)L}{T_s}\right).$$

· Then, $y_d[n]$ obtained by interpolating $x_d[m]$ with $\mathrm{sinc}(n/L)$ and rate $1/L$
has $Y_d(e^{\mathrm{j}2\pi f})$ the same as $Z_d(e^{\mathrm{j}2\pi f})$ by the relation Eq. (1).

- Motivations Revisited
  - Tx Part of Digital BB Modem
    - We want to generate an oversampled version of the complex envelope by **interpolating data sequence** $d[m]$ **with oversampled Tx pulse** $p(t)$.
      · $x_l(t) = \sum_{m=-\infty}^{\infty} d[m]p(t - mT)$
      · $x_{l,d}[n] = x_l(nT/M) = \sum_{m=-\infty}^{\infty} d[m]p_d[n-mM]$ with $p_d[n] = p(nT/M)$ being the M-times oversampled version of $p(t)$.
      · Take the real and the imaginary parts to have $x_{c,d}[n]$ and $x_{s,d}[n]$.

– Tx Part of Digital IF Modem

* If computational power and memory size allow, then we may directly generate $x_{\mathrm{IF},d}[n]$ as

$$x_{\mathrm{IF},d}[n] = \mathrm{Re}\left\{ \sum_{m=-\infty}^{\infty} d[m] p\left(\frac{nT}{LM} - mT\right) e^{\mathrm{j}2\pi f_{\mathrm{IF}} \frac{nT}{LM}} \right\}.$$

* If not, then we generate a high-rate sampled version of $x_l(t)$ from the low-rate sampled version $x_{l,d}[n]$ given by

$$x_{l,d}[n] = x_l(nT/M) = \sum_{m=-\infty}^{\infty} d[m] p_d[n - mM]$$

where $p_d[n] = p(nT/M)$.

- Upsample $x_{l,d}[n]$ by a factor of L and filter it with $r[n] = \mathrm{sinc}(n/L)$ to obtain $y_{l,d}[n]$.
- Multiply $y_{l,d}[n]$ element-by-element to $e^{\mathrm{j}2\pi f_{\mathrm{IF}} nT/(LM)}$.
- Take the real part to have $x_{\mathrm{IF},d}[n]$.