Complex Baseband Representation of Real-Valued Bandpass LTI Systems: Motivation

Prof. Joon Ho Cho
Department of Electrical Engineering, POSTECH

- Big Picture. 3 elements of communication systems
  - Tx
  - Channel: $\leftarrow$ We are here.
  - Rx

- Review. Complex baseband representation of real-valued bandpass signals
  - Why real?
  - Why bandpass?
  - Complex envelope
  - Quadrature vs. polar modulation
  - Homodyne vs. heterodyne up-conversion transmitters

- Preview. Often, a wireless channel can be well modeled by a real-valued LTI system. The key is to understand there are two cases to consider.

- Title Dissection
  - CT LTI (linear time-invariant) system
    * Def. A CT system $(\mathcal{X}, \mathcal{L}, \mathcal{Y})$ is LTI iff it is a CT system, i.e.,
      · $\mathcal{X}$: a set of all admissible CT inputs and
      · $\mathcal{Y}$: a set of all possible CT outputs, or that includes this set,

      and the mapping rule $\mathcal{L}$ from $\mathcal{X}$ into $\mathcal{Y}$ satisfies
      · the linearity (L) property, i.e., $x_1(t) \to y_1(t)$ and $x_2(t) \to y_2(t)$ implies
      $$a_1 x_1(t) + a_2 x_2(t) \to a_1 y_1(t) + a_2 y_2(t),$$
      · and the time-invariance (TI) property, i.e., $x(t) \to y(t)$ implies
      $$x(t - t_0) \to y(t - t_0).$$
    * Fig.

* Theorem. If $\mathcal{L}$ is a CT LTI system then there exists $h(t)$ such that

$$\begin{aligned} y(t) &= \mathcal{L}(x(t)) \\ &= \int_{-\infty}^{\infty} h(t-\tau)x(\tau)\mathrm{d}\tau \\ &= h(t) * x(t) \end{aligned}$$

where $h(t) \triangleq \mathcal{L}(\delta(t))$ is called the impulse response.

- $h(t)$ fully characterizes the system.
- 3 properties of the convolution integral, and parallel and serial interconnection of systems.
- Convolution in TD corresponds to multiplication in FD.
- ...

– Wireless channel as a CT LTI system with a bandpass impulse response

  * CT linear system model

    · CT system: When an EM wave is generated by a transmitter, it propagates a space and some part of the signal energy is picked up by an Rx antenna. Thus, **a wireless channel with a single Tx antenna and a single Rx antenna is well modeled by a CT system with a real-valued CT signal input and a real-valued CT signal output**.

    · Linear system: Since the EM wave is a wave, it propagates the space by following the superposition or linearity property. Since the received signal is a linear transform of the transmitted signal, **the channel is well modeled by a linear system**.

* CT LTI system model with real-valued bandpass impulse response
  - Often, but not always, the channel can be well modeled by an LTI system. In this case, there exists a real-valued impulse response $h(t)$ such that

$$y(t) = h(t) * x(t).$$

  - Q. Is $y(t)$ a real-valued bandpass signal? If so, why?

  - Q. Is it possible to model $h(t)$ by a real-valued bandpass signal? If so, why?

- Q. Suppose that $x(t)$ has the complex envelope $x_l(t)$ and that $y(t)$ has the complex envelope $y_l(t)$.
  Then, what is the relation between $x_l(t)$ and $y_l(t)$?
  How is the complex envelope $h_l(t)$ of the channel impulse response involved in the relation?
  That is, **find the relation among $x_l(t), h_l(t)$, and $y_l(t)$.**
- Fig.

∗ CT LTI specular multipath channel model

· Q. Is it necessary to have a real-valued bandpass impulse response $h(t)$? If not, why?

· Often, it is convenient to model the I/O relation of the wireless channel by

$$y(t) = \sum_{l=1}^{L} \alpha_l x(t - \tau_l),$$

which is called **a specular multipath channel model**.

· In this case, the impulse response is simply given by

$$h(t) = \sum_{l=1}^{L} \alpha_l \delta(t - \tau_l).$$

However, this impulse response is band-unlimited.

- Of course, we may bandpass filter this impulse response to have a band-limited impulse response.

- However, it is straightforward to see what happens in the channel when delta functions are used.
  α_l: gain of the lth path
  τ_l: propagation delay of the lth path

- Therefore, we carefully define **the complex baseband equivalent** to the channel impulse response in this case and use it instead of using the complex envelope. We still denote it by $h_l(t)$, which raises some confusion.

– Q. Find the relation among $x_l(t), h_l(t)$, and $y_l(t)$.

– A. We have two cases.

* Case 1:

$$y(t) = h(t) * x(t),$$

where $x(t), h(t)$, and $y(t)$ are **all real-valued bandlimited with same bandwidth**. $h_l(t)$ is **the complex envelope** of $h(t)$.

* Case 2:

$$y(t) = h(t) * x(t),$$

where $x(t)$ and $y(t)$ are real-valued bandlimited with same bandwidth, but $h(t)$ **is real-valued band unlimited and given by**

$$h(t) = \sum_{l=1}^{L} \alpha_l \delta(t - \tau_l).$$

$h_l(t)$ is **the complex baseband equivalent** to $h(t)$.