Image-Band Problem and IR Filter: Revisited

Prof. Joon Ho Cho
Department of Electrical Engineering, POSTECH

- Review. A heterodyne receiver has two LOs,
  - LO_1 used to down-convert an RF signal to an IF, and
  - LO_2 used to down-convert to the baseband.

  Before the down-conversion to the IF,
  - **an image band** must be rejected by **an IR filter**
  - if either interference or noise is present in the image band.

  Similar to upper-side tuning and lower-side tuning in heterodyne transmission,
  - there are **two LO frequencies** to choose in down-conversion to the IF, and
  - there are **two different image bands** in heterodyne reception.

- Preview. The **image-band problem in down-conversion to IF** is revisited. **By using all real-input real-output subsystems**, the problem and IR filtering are explained in the FD.

- Received Signal Model
  - The signal $y(t)$ received by an antenna consists in general of a desired signal, interference, and noise.

$$y(t) = \tilde{x}(t) + I(t) + N(t).$$

  - Fig. block diagram of Rx front end

  - Fig. received signal viewed in FD

- Down-conversion to IF with LO_1 frequency $f_c - f_{\mathrm{IF}}$ and image band centered at $\pm(f_c - 2f_{\mathrm{IF}})$
  - We've seen that, in this case, the IF signal is obtained by

$$y_{\mathrm{IF}}(t) = \mathrm{CS}\{\mathrm{IR}\,(y(t)) \cdot 2\cos(2\pi(f_c - f_{\mathrm{IF}})t + \phi_1)\}$$

    where the IR and the CS filters have real-valued impulse responses.
  - Fig. block diagram

– Let's take a look at the down-conversion operation carefully. Without IR filtering, we perform

$$\mathrm{CS}\left[y(t)\cdot\{e^{\mathrm{j}(2\pi(f_c-f_{\mathrm{IF}})t+\phi_1)}+e^{-\mathrm{j}(2\pi(f_c-f_{\mathrm{IF}})t+\phi_1)}\}\right].$$

– So, in the FD, we have

$$Y_{\mathrm{IF}}(f)=\mathrm{CS}\{Y(f-(f_c-f_{\mathrm{IF}}))e^{\mathrm{j}\phi_1}+Y(f+(f_c-f_{\mathrm{IF}}))e^{-\mathrm{j}\phi_1}\}.$$

– Fig. In FD, ...

- Notice that the first term moves
  * the signal located at $-f_c$ to $-f_{\text{IF}}$,
  * the signal located at f_c to $2f_c - f_{\text{IF}}$, and
  * the interference and noise located at $-(f_c - 2f_{\text{IF}})$ moves to f_{IF}.
- Notice that the second term moves
  * the signal located at f_c to f_{IF},
  * the signal located at $-f_c$ to $-2f_c + f_{\text{IF}}$, and
  * the interference and noise located at $f_c - 2f_{\text{IF}}$ to $-f_{\text{IF}}$.
- To remove the unwanted signal, interference, and noise,
  * we have to use an **IR filter** to reject the interference and noise **before mixing**, and
  * we have to use a **CS filter** to remove the high frequency signal and select the desired channel signal **after mixing**.
  * The stopband of the IR filter is located at $\pm(f_c - 2f_{\text{IF}})$ and have bandwidth ..., and
  * the passband of the CS filter is located at $\pm f_{\text{IF}}$ and have bandwidth ...

– The output IF signal is given by

$$y_{\mathrm{IF}}(t) = \mathrm{Re}\{(\tilde{x}_l(t)e^{-\mathrm{j}\phi_1})e^{\mathrm{j}2\pi f_{\mathrm{IF}} t}\}$$

when there is no in-band noise and all interference are filtered out. Thus, the complex envelope is $\tilde{x}_l(t)e^{-\mathrm{j}\phi_1}$.

– The quadrature demodulation paired with this IF down-conversion is given by the real and the imaginary parts of

$$\tilde{x}_l(t)e^{-\mathrm{j}(\phi_1+\phi_2)} = \mathrm{LPF}\left[y_{\mathrm{IF}}(t)\cdot 2e^{-\mathrm{j}(2\pi f_{\mathrm{IF}}t+\phi_2)}\right],$$

i.e.,

$$\mathrm{Re}\{\tilde{x}_l(t)e^{-\mathrm{j}(\phi_1+\phi_2)}\} = \mathrm{LPF}\left[y_{\mathrm{IF}}(t)\cdot 2\cos(2\pi f_{\mathrm{IF}}t+\phi_2)\right]$$

and

$$\mathrm{Im}\{\tilde{x}_l(t)e^{-\mathrm{j}(\phi_1+\phi_2)}\} = \mathrm{LPF}\left[y_{\mathrm{IF}}(t)\cdot(-\mathbf{2}\sin(2\pi f_{\mathrm{IF}}t+\phi_2))\right].$$

– Fig.

- Down-conversion to IF with LO_1 frequency $f_c + f_{\mathrm{IF}}$ and image band centered at $\pm(f_c + 2f_{\mathrm{IF}})$
  - We've seen that, in this case, the IF signal is obtained by

$$y_{\mathrm{IF}}(t) = \mathrm{CS}\{\mathrm{IR}\,(y(t)) \cdot 2\cos(2\pi(f_c + f_{\mathrm{IF}})t + \phi_1)\}$$

    where the IR and the CS filters have real-valued impulse responses.
  - Fig. block diagram

– Let's take a look at the down-conversion operation carefully. Without IR filtering, we perform

$$\mathrm{CS}\left[y(t)\cdot\{e^{\mathrm{j}(2\pi(f_c+f_{\mathrm{IF}})t+\phi_1)}+e^{-\mathrm{j}(2\pi(f_c+f_{\mathrm{IF}})t+\phi_1)}\}\right].$$

– So, in the FD, we have

$$Y_{\mathrm{IF}}(f)=\mathrm{CS}\{Y(f-(f_c+f_{\mathrm{IF}}))e^{\mathrm{j}\phi_1}+Y(f+(f_c+f_{\mathrm{IF}}))e^{-\mathrm{j}\phi_1}\}.$$

– Fig. In FD, ...

- Notice that the first term moves
  - the signal located at $-f_c$ to f_{IF},
  - the signal located at f_c to $2f_c + f_{\mathrm{IF}}$, and
  - the interference and noise located at $-(f_c+2f_{\mathrm{IF}})$ moves to $-f_{\mathrm{IF}}$.
- Notice that the second term moves
  - the signal located at f_c to $-f_{\mathrm{IF}}$,
  - the signal located at $-f_c$ to $-2f_c - f_{\mathrm{IF}}$, and
  - the interference and noise located at $f_c + 2f_{\mathrm{IF}}$ to f_{IF}.
- To remove the unwanted signal, interference, and noise,
  - we have to use an **IR filter** to reject the interference and noise **before mixing**, and
  - we have to use a **CS filter** to remove the high frequency signal and select the desired channel signal **after mixing**.
  - The stopband of the IR filter is located at $\pm(f_c+2f_{\mathrm{IF}})$ and have bandwidth ...
  - The passband of the CS filter is located at $\pm f_{\mathrm{IF}}$ and have bandwidth ...

– The output IF signal is given by

$$y_{\mathrm{IF}}(t) = \mathrm{Re}\{(\tilde{x}_l(t)e^{-\mathrm{j}\phi_1})e^{-\mathrm{j}2\pi f_{\mathrm{IF}}t}\} = \mathrm{Re}\{(\tilde{x}_l^*(t)e^{\mathrm{j}\phi_1})e^{\mathrm{j}2\pi f_{\mathrm{IF}}t}\}$$

when there is no in-band noise. Thus, the complex envelope is $\tilde{x}_l^*(t)e^{\mathrm{j}\phi_1}$.

– The quadrature demodulation paired with this IF down-conversion is given by

$$\mathrm{Re}\{\tilde{x}_l(t)e^{-\mathrm{j}(\phi_1-\phi_2)}\} = \mathrm{LPF}\left[y_{\mathrm{IF}}(t) \cdot 2\cos(2\pi f_{\mathrm{IF}}t + \phi_2)\right]$$

and

$$\mathrm{Im}\{\tilde{x}_l(t)e^{-\mathrm{j}(\phi_1-\phi_2)}\} = \mathrm{LPF}\left[y_{\mathrm{IF}}(t) \cdot \mathbf{2}\sin(2\pi f_{\mathrm{IF}}t + \phi_2)\right].$$

– Fig.