How to Handle Uncertainty in Electrical Signals: Random Variable

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- Big Picture. 3 elements of communication systems
  - Tx
  - Channel
  - Rx
- Are we done?
- Review. A received **real-valued bandpass** signal $y(t) = \tilde{x}(t) + I(t) + N(t)$ consists of
  - a channel-filtered linear-transformed version $\tilde{x}(t)$ of the transmitted signal,
  - a co-channel interference (CCI) or an adjacent channel interference (ACI) $I(t)$, and
  - a noise $N(t)$ often modeled by an **additive white Gaussian noise (AWGN)**.
- Preview. We will examine the AWGN component. The notion of the power spectral density (PSD) will also be useful in characterizing a transmitted signal in the FD.

- additive
  * The notion of statistical independence among random variables are reviewed.
- Gaussian
  * Real-valued and complex-valued Gaussian random variables are reviewed.
- noise
  * The concept of a random process is reviewed.
  * A temporal characterization of a random process is reviewed.
- white
  * A spectral characterization of a random process is reviewed.
  * The notion of the PSD is reviewed.
- real-valued bandpass random process
  * Complex baseband representation of real-valued bandpass random process
  * Complex baseband representation of real-valued band-limited white Gaussian noise

- Q. How to handle **uncertainty in electrical signals**?
- A. Model
  - real- or complex-valued scalar
  - vector
  - matrix
  - sequence
  - waveform
  - field
  - polynomial, graph, etc.

  with uncertainty as something
  - unknown and random
    * by providing **a probabilistic description** to the random quantity, or
  - unknown but not random
    * by giving **a parameterized structure** to the signal.

- Q. What is a real-valued random variable X?
- A. a real-valued scalar X whose uncertainty is described by **the cumulative distribution function (CDF)** $F_X(x)$ defined as

$$F_X(x) \triangleq \Pr(\{X \leq x\})$$

  - Fig. CDF

  - The CDF **fully characterize**s the random variable. $\equiv$ The probability of any event in terms of X can be computed from the CDF.

– **The probability density function (PDF)** $f_X(x)$ of a random variable X is defined as

$$f_X(x) \triangleq \frac{d}{dx} F_X(x),$$

or equivalently,

$$F_X(x) = \int_{-\infty}^{x} f_X(x) dx.$$

– Fig.

- A random variable is classified into
  - **a continuous random variable** if the CDF is a continuous function,
  - **a discrete random variable** if the CDF is a right-continuous piecewise constant function, or
  - **a mixed random variable** if the CDF is a weighted sum of a continuous and a piecewise constant function.
  - Q. What is the CDF and the PDF of each random variable?
    - (discrete) Rolling a die experiment. X represents the number shown.

    - (continuous) A uniform random variable on $[0, 1)$.

* Q. Given an input modeled by a Gaussian random variable with mean zero and variance σ^2, the system generates the output defined as $Y = \text{sign}(X)$. Find the CDFs and the PDFs of the output. What is the relation with a histogram?
* A.

- Q. What is a real-valued random vector $[X\ Y]^T$?
- A. a vector $[X\ Y]^T$ consisting of two real-valued random variables whose uncertainty is described by **the joint CDF** $F_{X,Y}(x, y)$ defined as

$$F_{X,Y}(x, y) \triangleq \Pr(\{X \leq x\} \cap \{Y \leq y\})$$

  - Fig. CDF

  - The joint CDF **fully characterize**s the random vector. $\equiv$ The probability of any event in terms of $[X\ Y]^T$ can be computed from the joint CDF.

- **The joint PDF** $f_{X,Y}(x,y)$ of a random vector $[X\ Y]^T$ is defined as

$$f_{X,Y}(x,y) \triangleq \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y),$$

or equivalently,

$$F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(x,y) dy dx.$$

- Q. What is the joint CDF and the joint PDF of each random vector?
  * Tossing a coin two times. Define X and Y as ...
  * Tossing a coin once. Define X and Y as ...
- A.

- Q. What is the **difference between** $\{X = Y\}$ **and** $X = Y$?
- A. an event vs. the equality of two random variables

- Q. How to **probabilistically model a complex-valued scalar signal**?
- A. A complex-valued random variable Z can be viewed as a length-2 real-valued random vector consisting of two real-valued random variables
  - $X \triangleq \mathrm{Re}\{Z\}$ and
  - $Y \triangleq \mathrm{Im}\{Z\}$.

- Q. How to probabilistically model more than two real-valued scalar signals or more than one complex-valued scalar signals?
- A. By extending the notion of the joint CDF and the joint PDF ...