Lec. 03

2013년 3월 5일 화요일

오후 2:52

Review

- single-source single-destination
  - data compression
  - data transmission

- Data Compression

$$\text{source} \xrightarrow{X} \text{encoder} \xrightarrow{Y} \text{decoder} \xrightarrow{\hat{X}} \text{sink}$$

Two perf. metrics
  - compression rate
  - error rate

- Data Transmission

$$\text{source} \rightarrow \text{encoder} \rightarrow \text{channel} \rightarrow \text{decoder} \rightarrow \text{sink}$$

Two perf. metrics
  - transmission rate
  - error rate

page 01

2013년 3월 7일 목요일
오후 2:10

- error rate = 0 ← Simplify !

Zero-error data compression
$\Updownarrow$ ← asymptotically zero error...
non-zero-error " "

- Source $\xrightarrow{X_i}$ 　　X_i's i.i.d.
X_i discrete
$\mathcal{X}$ finite
↑ source alphabet

해석학
$(x_n)_n$
$(A_n)_n \sim (B_n)_n$
$(f_n(x))_n$

x_i	$\Pr(X_i = x_i)$
a	1/4
b	1/4
c	1/2

- $x_i \in \mathcal{X}$ → encoder → $\mathcal{D}^*$ variable length

$\mathcal{D}$ ← code alphabet 　 $|\mathcal{D}| = D < \infty$

codeword ∈ $\mathcal{D}^*$ 　 $|\mathcal{D}^*| = \infty$.

p_i	x_i	$e(x_i)$	$l(x_i)$
1/4	a	0	1
1/4	b	10	2
1/2	c	11	2

a v.l. source code

- average codeword length

Given $\mathcal{C}$, the averge codeword length $L(\mathcal{C})$ is defined as

$$\boxed{L(\mathcal{C})} = E\{l(X)\} = \sum_{i=1}^{|\mathcal{X}|} l(x_i) p_i$$

- Change of base

$$2^{100} = 4^{50}$$

base, exponent

commonality
difference

- The Law of Large Numbers

" arithmetic average ≃ statistical average "

$$\frac{X_1 + X_2 + \cdots + X_N}{N} \simeq E\{X_n\}$$

$$\frac{X_1+X_2+\cdots+X_N}{N} \simeq \underline{E\{X_n\}}$$ "average"

when $N \gg 1$.

strong

- modes of convergence
  - almost sure
  - in the pth mean
  - in probability
  - in distribution

weak

$$X_n \xrightarrow{a.s.} X$$

a.s. $\Rightarrow$ in p. $\Leftarrow$ LP

$\Downarrow$

in d.

- Central Limit TheoremS.

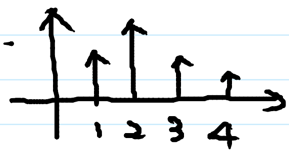
$$\frac{X_1+X_2+\cdots+X_N}{\sqrt{N}} \xrightarrow{\text{in distribution}} \underline{N(0,\sigma^2)}$$

X_i uncorrelated
mean zero
variance σ^2

- Chebychev Inequality

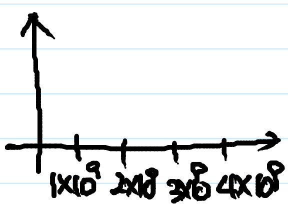
X, $E[X]=\mu$, $Var[X]=\sigma^2>0$

$$Pr(|X-\mu| \geq k) \leq \frac{\sigma^2}{k^2}$$



$1.5E[\ell(x)]$

X_1 $E[\ell(x)]$



$\begin{bmatrix} X_1 \\ \vdots \\ X_{10^9} \end{bmatrix}$ $NE[\ell(x)]$

$1.5NE[\ell(x)]$

$$Pr\left(\left|\sum_{i=1}^{N} X_i - N\mu\right| > 1.5N\mu\right) \leq \underline{\qquad\qquad}$$

$$= Pr\left(\left|\sum_{i=1}^{N}(X_i-\mu)\right| > 1.5N\mu\right) \qquad E\left\{\left(\sum_{i=1}^{N}(X_i-\mu)\right)^2\right\}$$

$$= Pr\left(\left(\sum (X_i-\mu)\right)^2 > (1.5N\mu)^2\right) \leq \frac{N\sigma^2}{\quad} = \frac{\sigma^2}{\quad} \to 0$$

$$= \Pr\left(\left(\sum_{r=1} \ldots\right) > (1.5N\mu)\right) \le \frac{\ldots}{(1.5N\mu)^2} \ldots \frac{\ldots}{(1.5)^2 N\mu^2} \ldots$$

$N=1$

$$\Pr\left(|X-\mu| > 1.5\mu\right) \le \frac{\sigma^2}{(1.5)^2\mu}$$

page 02

2013년 3월 6일 수요일

오후 9:08

- objective function $L(\mathcal{C})$

$$\boxed{\begin{array}{l} \min_{\mathcal{C}} L(\mathcal{C}) \\ \text{subject to zero error} \end{array}}$$

"Given source distribution"

subject to zero error ⟶ i.e. $\mathcal{C} \in \Omega$. ← constraint set

$$\Longleftrightarrow \min_{\mathcal{C}} L(\mathcal{C})$$

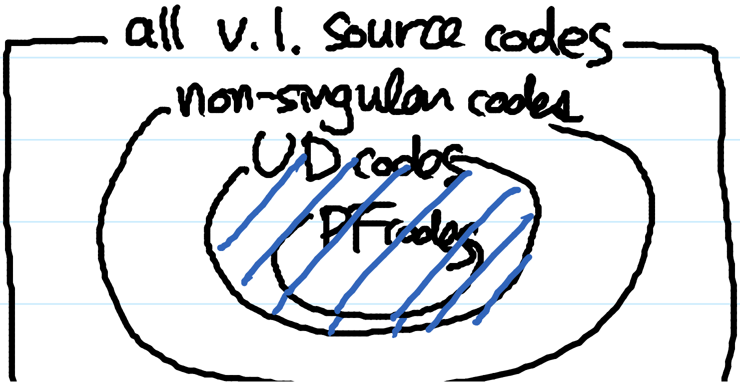
cf) such that

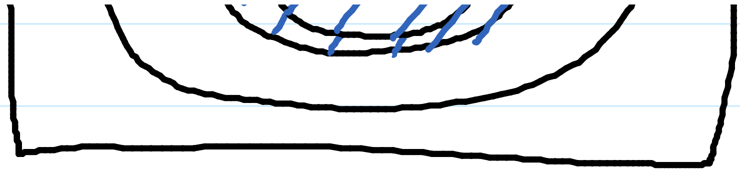
s.t. $\mathcal{C}$ is a UD variable-length source code.

- set of PF codes $\subset$ set of UD codes

$\not\supset$

x_i	$c(x_i)$
a	$\boxed{0}$
b	01
c	011

- 



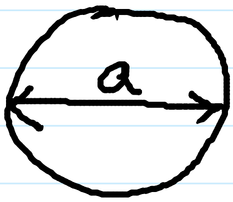
page 03

2013년 2월 7일 목요일

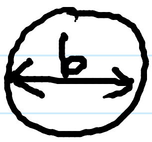
오후 3:12

$$3 < x < 4$$

$$3 < \pi < 4$$

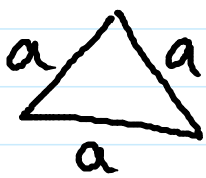


circumference a'

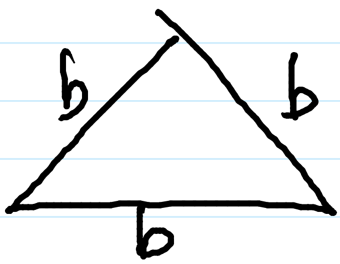


b'

$$a : b = a' : b'$$

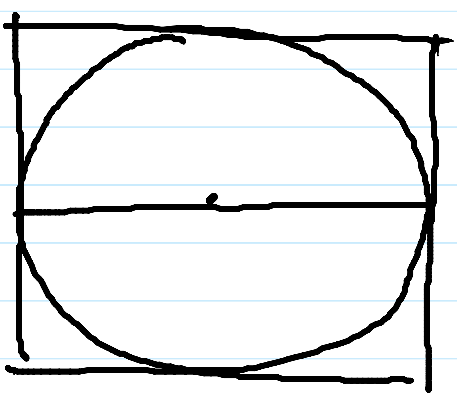


$3a$



$3b$

$$a : b = 3a : 3b$$



$$2 < \pi < 4$$

$$a_n < \pi < b_n$$

3.1415 ...

a lower bound

an upper bound

$$H_D(X) < \min_{\mathcal{C} \in \mathrm{BUD}} L(\mathcal{C}) < H_D(X) + 1$$