Lec. 04

2013년 3월 12일 화요일

오후 2:04

Review

single-source single-destination

- data compression ←
- data transmission

Two performance metrics

Zero-error data compression

Problem formulation

objective function

$$\min_{\mathcal{C}} \; L(\mathcal{C})$$

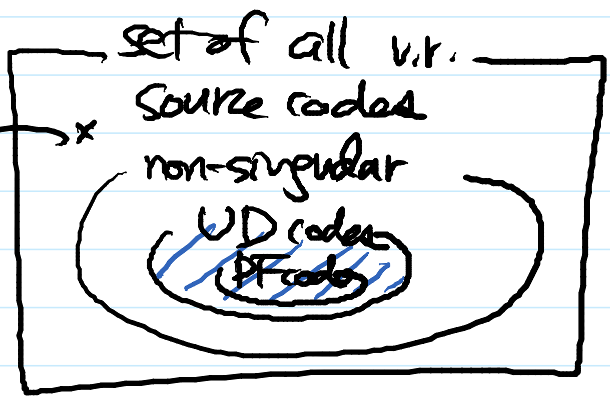
subject to $\mathcal{C}$ is uniquely decodable.

$$\mathcal{C} \in \cancel{UD}\,\Omega$$

constraint set.

table

x_i	$c(x_i)$
1	....
2	....
:	..
$\|\mathcal{X}\|$	..



lower bound $\le \min L(\mathcal{C}) <$ upper bound

G isUD

x_i	$c(x_i)$
1	$\underline{c}_1$
2	$\underline{c}_2$
⋮	⋮
$\lvert X \rvert$	$\underline{c}_{\lvert X \rvert}$

If $\mathcal{C}$ is UD, then ......

$$H_D(X) \le L(\mathcal{C})$$

the entropy of X

D-ary finite-length vectors

N times use of the V.L. source code.

Example. representation visualization

x_i	$c(x_i)$	$l(x_i)$
1	0	1
2	10	2
3	11	2

N times

N ~ 2N

A_N 1 ....

abstract

concrete

$$(x^1+x^2+x^2)^2 = x^1\cdot x^1 + x^1\cdot x^2 + x^1\cdot x^2 + x^2\cdot x^1 + x^2\cdot x^2 + x^2\cdot x^2 + x^2\cdot x^1 + x^2\cdot x^2 + x^2\cdot x^2$$

$$= A_2^{(2)} x^2 + A_3^{(2)} x^3 + A_4^{(2)} x^4$$

$A_2^{(2)} = 1$, $A_3^{(2)} = 4$, $A_4^{(2)} = 4$.

length enumerator

$$(x^{l_1} + x^{l_2} + \cdots + x^{l_{\lvert X \rvert}})^N = \sum^{N\cdot l_{max}} A_i^{(N)} x^i$$

$$\boxed{(x^{l_1}+x^{l_2}+\cdots+x^{l_{|X|}})^N = \sum_{i=N\cdot l_{min}}^{N\cdot l_{max}} A_i^{(N)} x^i}$$

Q. How many distinct codeword sequences of length i?

A. If $\mathcal{C}$ is UD, then

$$\boxed{A_i^{(N)} \le D^i \quad \text{for all } N\cdot l_{min} \le i \le N\cdot l_{max}}$$

$D=2$.

$i=4$

$$\left(\sum_{i=1}^{|X|} x^{l_i}\right)^N = \sum_{i_N=1}^{|X|}\cdots\sum_{i_2=1}^{|X|}\sum_{i_1=1}^{|X|} x^{l_{i_1}+l_{i_2}+\cdots+l_{i_N}}$$

$$= \sum_{i=N l_{min}}^{N l_{max}} A_i^{(N)} x^i$$

Choose $x=D^{-1}$!

$$\left(\sum_{i=1}^{|X|} D^{-l_i}\right)^N = \sum_{i=N\cdot l_{min}}^{N\cdot l_{max}} A_i^{(N)} D^{-i} \qquad \le D^i$$

$$\le \sum_{i=N\cdot l_{min}}^{N\cdot l_{max}} 1$$

$$= N\cdot l_{max} - N\cdot l_{min} + 1$$

$$\le N\cdot l_{max}, \quad \forall N.$$

Two directions

$\begin{cases} N \downarrow \\ N \uparrow \end{cases}$

$$\sum_{i=1}^{|\mathcal{X}|} D^{-l_i} \leq \underbrace{(N \cdot l_{max})^{\frac{1}{N}}}, \forall N.$$

$$= e^{\frac{1}{N}\ln(N \cdot l_{max})}$$

$$\boxed{\lim_{x \to x_0} f(x) = f(\lim_{x \to x_0} x) = f(x_0)}$$ continuity.

$$a^b = e^{b \ln a}$$

$$= e^{\ln a^b} = e^{\log_e a^b} = a^b$$

$$\lim_{N \to \infty} \frac{1}{N} \ln(N \cdot l_{max}) = \lim_{N \to \infty} \frac{\ln(N \cdot l_{max})}{N}$$

$$= \lim_{N \to \infty} \frac{\ln N + \ln l_{max}}{N}$$

$$= 0$$

If $\mathcal{C}$ is UD, then

$$\sum_{i=1}^{|\mathcal{X}|} D^{-l_i} \leq 1$$

Kraft's inequality

To show,

if $\mathcal{C}$ is UD, then

$$\underbrace{H_D(X)}_{\text{entropy of } X.} \le L(\mathcal{C})$$

The entropy lower bound

$$\Rightarrow H_D(X) \le \min_{\mathcal{C}\ \mathrm{UD}} L(\mathcal{C})$$

Def. Given a discrete r.v. X,

$$H_D(X) = \sum_{x \in \mathcal{X}} p(x) \log_D \frac{1}{p(x)}$$

D-ary source

Too good to be true!