Lec. 07

2013년 3월 20일 수요일

오후 9:28

Review

- data compression
  - zero-error — variable-length
  - fixed-length
- neither zero-error nor non-zero error
  - $x=0$ xor $x \neq 0$
  - $x_n \neq 0, \forall n$ $\quad x_n \to 0$ as $n \to \infty$

  tends to

- Def. A codeword symbols per source symbol R is achievable if there exists a sequence of (fixed-length) codes $\mathcal{C}_K$ s.t.

$$R_K \to R \quad \text{and}$$

$$P_{e,K} \to 0 .$$

Page 01

2013년 3월 20일 수요일

오후 9:28

Start from what you know.

the simplest. ⇐ Simplify!

discrete optimization 이산최적화

(continuous) optimization (연속)최적화

mixed "

convergence of a sequence of

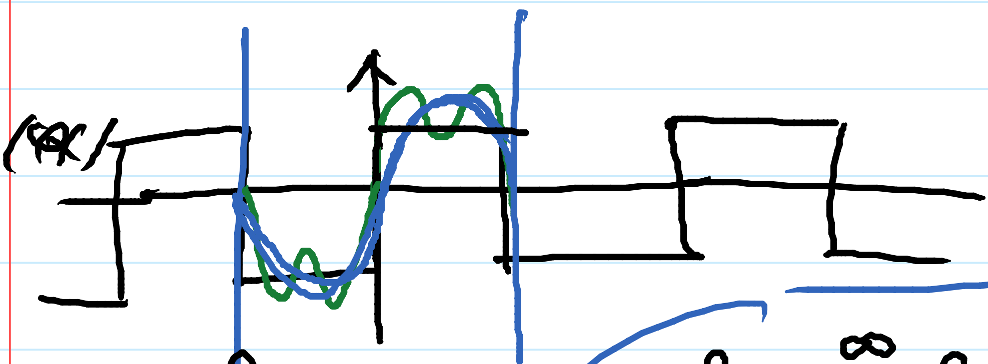
(i) real numbers

(ii) real functions

measurable

(iii) random variables.

(ii)



uniform

⇓

everywhere

⇓

almost everywhere

L^p convergence

in measure

$$f(x) = \sum_{n=1}^{\infty} f_n(x)$$

$$\int_a^b f(x)\,dx \overset{?}{=} \sum_{n=1}^{\infty} \int_a^b f_n(x)\,dx$$

2013년 3월 20일 수요일
오후 9:28

(iii) Convergence of a sequence of random variables

4 modes of convergence

almost sure L^p (convergence in the p-th mean)

$$\begin{array}{ccc} \text{almost sure} & & L^p \\ & \searrow\!\!\Rightarrow \quad \Leftarrow\!\!\swarrow & \\ & \text{in prob.} & \\ & \Downarrow & \\ & \text{in distribution} & \end{array}$$

strong

weak

" $\Downarrow$ diagram "

Convergence in probability

Def. Given a probability space $(\Omega, \mathcal{F}, P)$, a sequence of random variables X_n converges in probability to a random variable X

if $\forall \varepsilon > 0$ $\Pr(|X_n - X| > \varepsilon) \to 0$ as $n \to \infty$.

cf) $$m\left(\left\{\omega : |X_n(\omega) - X(\omega)| > \varepsilon\right\}\right) \to 0$$

Notation $X_n \xrightarrow{P} X$ or $X_n \to X$ p.

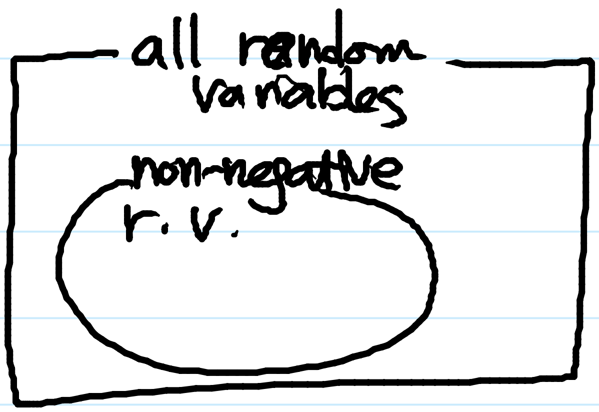
Page 03

2013년 3월 20일 수요일

오후 9:28

- Markov's inequality
  Chebychev's "
  WLLN

- The Markov Inequality

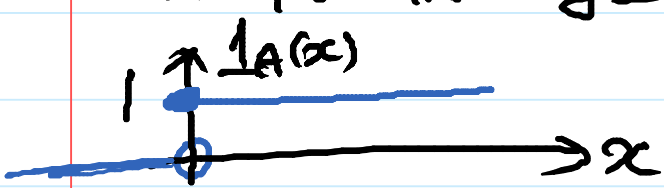


Def. A random variable X is non-negative if $Pr(X \geq 0) = 1$

(MI) If X is non-negative, then $Pr(X > a) \leq \frac{E[X]}{a}$.
where $a > 0$ is a const.

- Indicator function $1_A(x) \triangleq \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A. \end{cases}$

$A = \{y \in \mathbb{R} : y \geq 0\}$

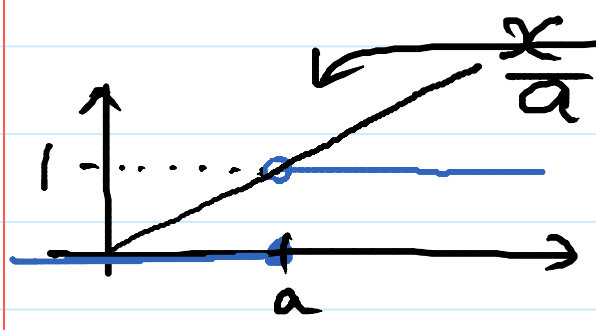


- $$\Pr(X\in A) = E\{1_A(X)\} = \sum_{x\in\Omega} 1_A(x)\Pr(X=x) = \sum_{x\in A}\Pr(X=x) = \Pr(X\in A)$$

$$\Pr(X>a) = E\{1_{\{x\in\mathbb{R}:\, x>a\}}(X)\}$$

- $f_1(x) \geq f_2(x)$, $x\in A$ and $P(X\in A)=1$

$\Rightarrow E[f_1(X)] \geq E[f_2(X)]$

- 

$1_A(x) \leq \frac{x}{a}$, $\forall x\geq 0$

$\Rightarrow \Pr(X>a) \leq \frac{E[X]}{a}$.

- The Chebychev Inequality

$$\Pr(|X-\bar{X}|>k\sigma) \leq \frac{1}{k^2}$$

(proof)

$$= \Pr((X-\bar{X})^2 > k^2\sigma^2) \leq \frac{E(X-\bar{X})^2}{k^2\sigma^2} = \frac{1}{k^2}$$

page 05

2013년 3월 20일 수요일

오후 10:49

The Weak Law of Large Numbers

Theorem

Given a sequence of uncorrelated r.v.'s X_n with $E[X_n]=\mu$ and $Var[X_n]=\sigma^2<\infty$, the sequence of random variables $(Y_n)_{n=1}^{\infty}$ satisfies

$$Y_n \to \mu \text{ in prob.}$$

if

$$Y_n \triangleq \frac{1}{n}(X_1+X_2+\cdots+X_n).$$

2 sequences
$$\begin{cases} X_1 & X_2 & X_3 & \cdots \\ Y_1 & Y_2 & Y_3 & \cdots \end{cases}$$

(proof)

To show

$$\forall \varepsilon>0,\ \Pr(|Y_n-\mu|>\varepsilon)\to 0 \text{ as } n\to\infty.$$

$$\Pr\left(\left|\frac{X_1+X_2+\cdots+X_n}{n}-\frac{n\mu}{n}\right|>\varepsilon\right)$$

$$=\Pr\left(\left|\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)\right|>\varepsilon\right)$$

$$\le \frac{E\left\{\frac{1}{n^2}\left(\sum_{i=1}^{n}(X_i-\mu)\right)^2\right\}}{\varepsilon^2} = \frac{\sum_{i=1}^{n}E\{(X_i-\mu)^2\}}{n^2\varepsilon^2}=\frac{\sigma^2}{n\varepsilon^2}$$

(WLLN) $$\frac{X_1+X_2+\cdots+X_n}{n}\to E\{X_i\} \text{ in prob.}$$

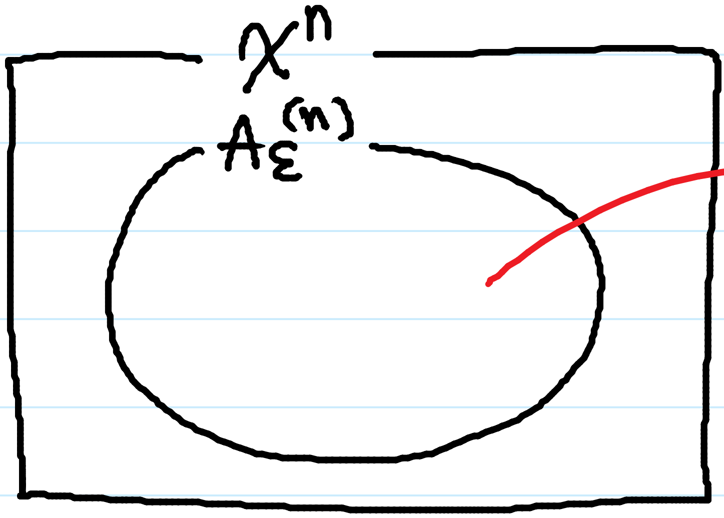
(WLLN) $\frac{X_1+X_2+\cdots+X_n}{n} \rightarrow E\{X_i\}$ in prob.

page 06

2013년 3월 20일 수요일

오후 11:00

- Preview



$Pr(\underline{X}^{(n)} \notin A_\varepsilon^{(n)}) < \varepsilon^{(n)}$

$|A_\varepsilon^{(n)}| < |\mathcal{X}^n|$

WLLN!