Lec. 09

2013년 3월 28일 목요일

오후 1:50

data < compression / transmission

Review

- The source coding theorem

  a sequence of fixed-length source codes $\mathcal{C}^{(k)}$

  s.t. code rate $R^{(k)} \to R$ [code symbols/source symbol]

  error rate $P_e^{(k)} \to 0$,

  as $k \to \infty$

  an achievable rate

- Direct Part of source coding theorem

  If $R \geq H(X)$, then R is achievable

- Converse of source coding theorem

  If R is achievable, then $R \geq H(X)$.

  $\Leftrightarrow$ If $R < H(X)$, then R is not achievable.

  strong converse

  weak converse

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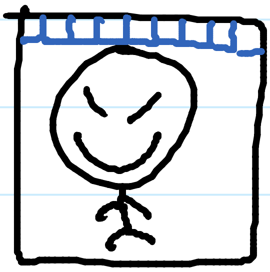


- Direct Part.

variable-length source coding

1st lesson : stack the outputs of the DMS and compress.

2nd lesson : consider the limit

not intuitive

counter-intuitive.

fixed-length source coding

WLLN: $\frac{1}{n}\left(\sum_{i=1}^{n} -\log P_X(X_i)\right) \to H(X)$ in prob.

$A_\varepsilon^{(n)} \triangleq \left\{ (x_1, \cdots, x_n) : \left| -\frac{1}{n} \log p(x_1, x_2, \cdots, x_n) - H(X) \right| < \varepsilon \right\}$

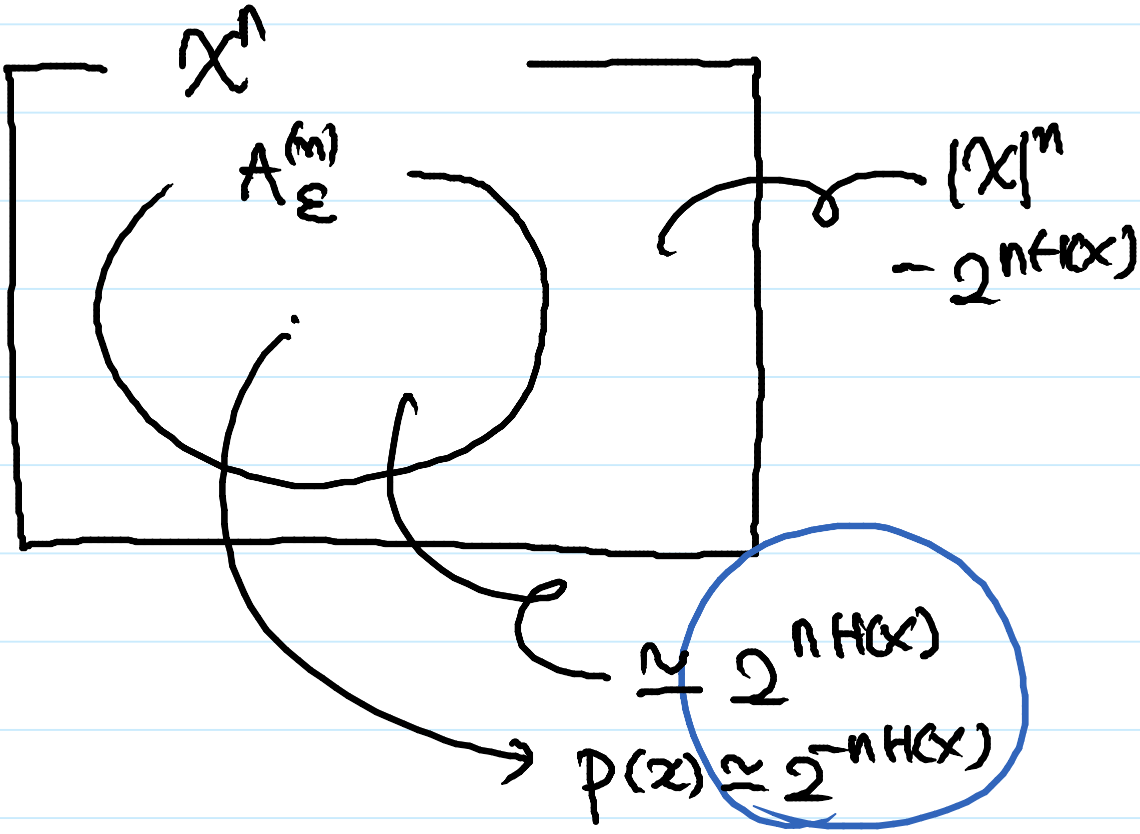
$P(A_\varepsilon^{(n)}) > 1-\varepsilon$ for sufficiently large n

$|A_\varepsilon^{(n)}| > (1-\varepsilon) 2^{n(H(X)-\varepsilon)}$ f.s.l. n

$|A_\varepsilon^{(n)}| < 2^{n(H(X)+\varepsilon)}$

$2^{-n(H(X)+\varepsilon)} \le p(x_1, x_2, \cdots, x_n) \le 2^{-n(H(X)-\varepsilon)}$

AEP (asymptotic equi-partition property)



- $P_c \triangleq \Pr(\underline{X}^{(n)} \in \mathcal{C}^{(n)})$

$$= \sum_{\underline{x}^{(n)} \in \mathcal{C}^{(n)}} \Pr(\underline{X}^{(n)} = \underline{x}^{(n)})$$

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(Metrics)

- **Shannon's Information Measures**

$$H(X) = -\sum (\log p_i)\, p_i = E\left\{\log \frac{1}{P_X(X)}\right\}$$

$$H(X,Y) = -\sum_x \sum_y (\log P_{xy})\, P_{xy} = E\left\{\log \frac{1}{P_{X,Y}(X,Y)}\right\}$$

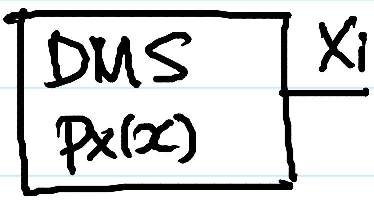
$$\vdots$$

$$H(X_1, X_2, \cdots, X_n) = E\left\{\log \frac{1}{P_{X_1 X_2 \cdots X_n}(X_1, X_2, \cdots, X_n)}\right\}$$

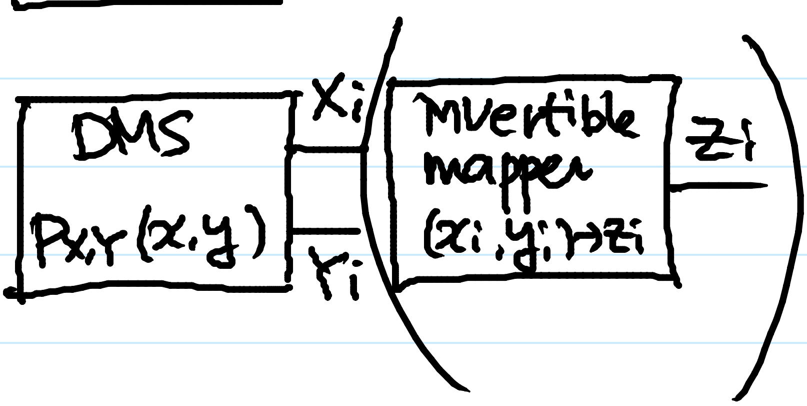
x	$\Pr(X=x)$
1	p_1
2	p_2
$\vdots$	$\vdots$
$\lvert X \rvert$	$p_{\lvert X \rvert}$

$x \backslash y$	y_1	y_2
x_1	p_{11}	p_{12}
x_2	p_{21}	p_{22}

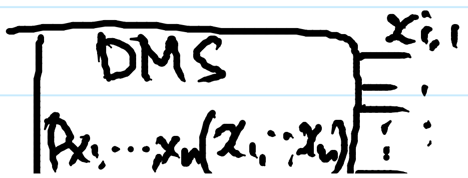
- **Physical Meaning of Entropy**



$H_2(X)$ [bits/source symbol]



$H_2(X,Y)$



$H_2(X_1, X_2, \cdots, X_n)$

$P_{X_1 \cdots X_n}(x_1, \cdots, x_n)$ $\vdots$ $x_{i,n}$

$(x_1, x_2, \ldots, x_n)$

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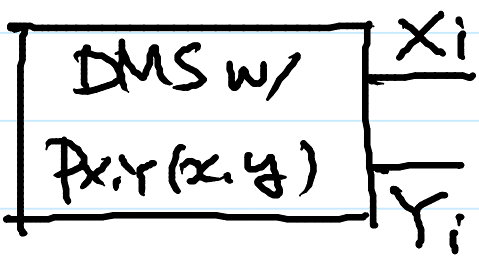
- Properties of Entropy
  - $H(X) \geq 0$ equality holds iff X is deterministic.
  - $H(X) \leq \log |X|$ equality holds iff X is uniform.
  - $H_a(X) < H_b(X)$ if $a > b$
  - $H_a(X) = H_b(X) \dfrac{\log b}{\log a}$
  - $Y = f(X)$ where f is one-to-one correspondence

    $\Rightarrow H(X) = H(Y)$

- Conditional Entropy



X_i: $\boxed{0}\,\boxed{0}\,1\,\boxed{0}\,1\,1\,\boxed{0}\,\boxed{0}\,\boxed{0}\,1\,\boxed{0}\,1\,1 \cdots$

Y_i: $\boxed{a}\,\boxed{a}\,b\,\boxed{b}\,a\,b\,\boxed{b}\,\boxed{a}\,\boxed{a}\,b\,\boxed{b}\,b\,a \cdots$

Q. How many bits/Y_i symbol when $X_i = 0$?

A ① $H(Y)$ ② $\sum_{y \in \mathcal{A}} P_{Y|X}(y|x=0)$

$\times \log 1/P_{Y|X}(y|x=0)$

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Entropy of Y given $\{X=0\}$

$$H(Y|\{X=0\}) = H(Y|X=0)$$

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