Lec. 10

2013년 4월 2일 화요일

오후 1:52

- Review

- structure of this course

  zero-error data compression

  source coding theorem

  Shannon's information measures

  channel coding theorem

  Rate distortion theory

  P_b vs. E_b/N_0

  KL distance and detection theory

- Entropy

r. variables: discrete, continuous, mixed

(i) $H(X) = -\sum_{x \in \mathcal{X}} \Pr(X=x) \log \Pr(X=x)$

representation.

$$= -\sum_{i=1}^{|\mathcal{X}|} p_i \log p_i$$

interpretation.

$$= \cdots$$

$$= E\left\{\log \frac{1}{P_X(X)}\right\}$$

Physical meaning $\boxed{\begin{matrix}\text{DMS}\\ \text{w/ } P_X\end{matrix}} \xrightarrow{X_i}$

(ii) $H(X,Y) = E\left\{\log \frac{1}{P_{X,Y}(X,Y)}\right\}$

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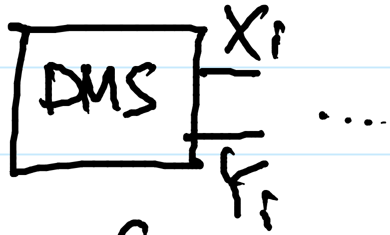
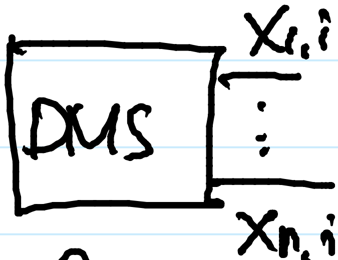
Physical meaning. 

(iii) $H(X_1, X_2, \cdots, X_n) = E\left\{\log \frac{1}{P_{X_1,\cdots,X_n}(X_1,\cdots,X_n)}\right\}$

Physical meaning 

(iv) Conditional entropy of X given $\{Y=y\}$
Entropy of X conditioned on $\{Y=y\}$

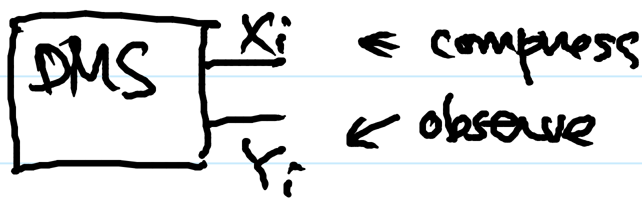
$$H(X|Y=y) = \sum_{x\in\mathcal{X}} P_{X|Y=y}(x)\log\frac{1}{P_{X|Y=y}(x)}$$

$$= \sum_{x\in\mathcal{X}} P_{X|Y}(x|y)\log\frac{1}{P_{X|Y}(x|y)}$$

$$\overset{?}{=} E\left\{\log\frac{1}{P_{X|Y}(X|y)}\right\}$$

$$= E\left\{\log\frac{1}{P_{X|Y}(X|Y)}\,\middle|\,Y=y\right\}$$

$$= E\left\{\log\frac{1}{P_{X|Y}(X|y)}\,\middle|\,Y=y\right\}$$

Physical meaning 

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(V) Conditional entropy of X given Y

= Entropy of X conditioned on Y.

$H(X|Y)$

$$= \sum_{y \in \mathcal{Y}} H(X|Y=y) P_Y(y)$$

$$\neq E[H(X|Y=Y)]$$

$$\neq E[H(X|Y)]$$

$$= \boxed{E\left\{\log \frac{1}{P_{X|Y}(X|Y)}\right\}}$$

$$= \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} P_{X,Y}(x,y) \log \frac{1}{P_{X|Y}(x|y)}$$

$$= \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} P_{X|Y}(x|y) P_Y(y) \log \frac{1}{P_{X|Y}(x|y)}$$

$$= \sum_{y \in \mathcal{Y}} \left(\sum_{x \in \mathcal{X}} P_{X|Y}(x|y) \log \frac{1}{P_{X|Y}(x|y)} \right) P_Y(y)$$

$$= \sum_{y \in \mathcal{Y}} H(X|Y=y) P_Y(y)$$

cf) Total expectation thm.

$E\{X\}$

$= E[E\{X|Y\}]$

$E[X|Y=y]$: deterministic scalar

$E[X|Y]$: r.v.

$$Y \to \boxed{E[X|Y=y]} \to E[X|Y]$$

$$\sum_{y \in \mathcal{Y}} E[X|Y=y] P_Y(y)$$

cf) $$Y \to \boxed{g(y)} \to g(Y)$$

$$E[g(Y)] = \sum_{y \in \mathcal{Y}} g(y) P_Y(y)$$

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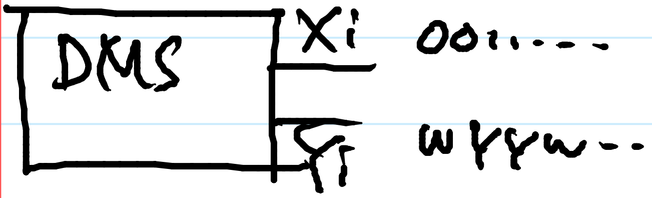
Physical meaning



$$H(X|Y=y)Pr(y) + H(X|Y=w)Pr(w) = H(X|Y)$$

- Def. of conditional prob.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Total prob. Thm.

$$P(A) = \sum_i P(A|B_i)P(B_i)$$

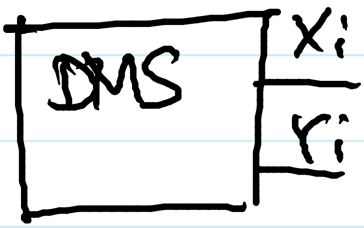
- Bayes' theorem

$$P(B_j|A) = \cdots$$

· Chain rule

$$\underbrace{H(X,Y)}_{(X_i,Y_i) \to Z_i} = H(X) + H(Y|X)$$

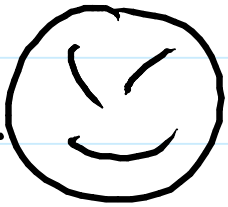
$$= H(Y) + H(X|Y)$$



compress (X,Y)

compress Y then compress X

Guess



$$\le H(X) + H(Y)$$

compress X then compress Y.

(vi) $H(X,Y|Z) = E\left\{\log \frac{1}{P_{X,Y|Z}(X,Y|Z)}\right\}$

$= H(X|Z) + H(Y|X,Z)$

$= \dots = H(X|Z) + H(Y|X,Z)$ $(X;Y|Z$ $(X;Y|Z)$ $)$ $)$

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(vi) $I(X;Y) = H(X) - H(X|Y)$

$= H(Y) - H(Y|X)$

$= I(Y;X)$

≥ 0