Lec. 12

2013년 4월 4일 목요일

오후 2:24

Review

- The KL distance from p_X to p_Y

$$D(p_X \| p_Y) \triangleq E\left\{\log \frac{p_X(X)}{p_Y(X)}\right\}$$

$$= E\left\{\log \frac{1}{p_Y(X)}\right\}$$ ← mismatched. not minimum...

$$- E\left\{\log \frac{1}{p_X(X)}\right\}$$ ← minimum description length

- "Shannon Code": a variable-length source code, a PF code, $l_i = \left\lceil \log_D \frac{1}{p_i} \right\rceil$

- $D(p_X \| p_Y) \geq 0$ where the equality holds iff $p_X = p_Y$.

- $$D(p_{X,Y} \| p_{Z,W}) = E\left\{\log \frac{p_{X,Y}(X,Y)}{p_{Z,W}(X,Y)}\right\}$$

$$= D(p_X \| p_Z) + \underbrace{D(p_{Y|X} \| p_{W|Z})}_{= E\left\{\log \frac{p_{Y|X}(Y|X)}{p_{W|Z}(Y|X)}\right\}}$$

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- $I(X;Y) = E\left\{ \log \frac{P_{XY}(X,Y)}{P_X(X)P_Y(Y)} \right\}$

  $= D(P_{X,Y} \,\|\, P_X P_Y)$

  ≥ 0 where the equality holds iff X & Y are independent.

- $I(X;Y|Z) \geq 0$ where the equality holds X & Y are conditionally independent given Z

  $\left(P_{X,Y|Z}(x,y|z) = P_{X|Z}(x|z) \cdot P_{Y|Z}(y|z) \quad \forall x \forall y, \forall z \right)$

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- Fano's inequality

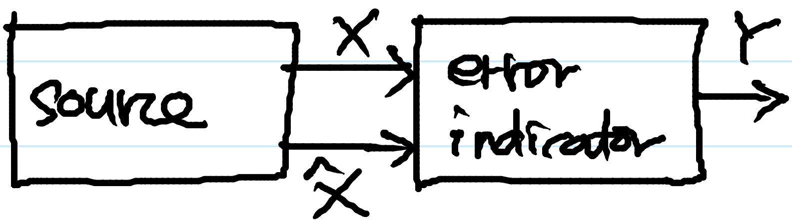
  Given two random variables X & $\hat{X}$ w/ the same sample space $\mathcal{X}$, the Fano's inequality rlnts

$$P_e \triangleq \Pr(X \neq \hat{X}) \quad \& \quad H(X|\hat{X})$$

in such a way that

$$\boxed{H(X|\hat{X}) \leq h_b(P_e) + P_e \cdot \log(|\mathcal{X}|-1)}$$

$$\leq 1 + P_e \log|\mathcal{X}|$$

- 

- $$H(X|\hat{X}) = H(X|\hat{X}) + H(Y|X,\hat{X})$$

$$= H(X,Y|\hat{X})$$

$$= H(Y|\hat{X}) + H(X|Y,\hat{X})$$

$$\leq h_b(P_e) + \sum_{y=0}^{1}\sum_{\hat{x}} H(X|Y=y,\hat{X}=\hat{x}) \Pr(Y=y,\hat{X}=\hat{x})$$

$$= h_b(P_e) + \sum_{\hat{x}} H(X|Y=1,\hat{X}=\hat{x}) \Pr(Y=1,\hat{X}=\hat{x})$$

$\sum_x \sum_y = \sum_y \sum_x$

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$$\leq h_b(P_e) + \sum_{\hat{x}} \log(|X|-1) \Pr(Y=1, \hat{X}=\hat{x})$$

$$= h_b(P_e) + \underbrace{\boxed{\sum_{\hat{x}} \Pr(Y=1, \hat{X}=\hat{x})}}_{=\Pr(Y=1)} \cdot \log(|X|-1)$$

$$= h_b(P_e) + P_e \cdot \log(|X|-1)$$

· Implications

명제 (i) If $P_e \to 0$, then $H(X|\hat{X}) \to 0$

대우 (ii) If $H(X|\hat{X}) \not\to 0$, then $P_e \not\to 0$

↳ contrapositive

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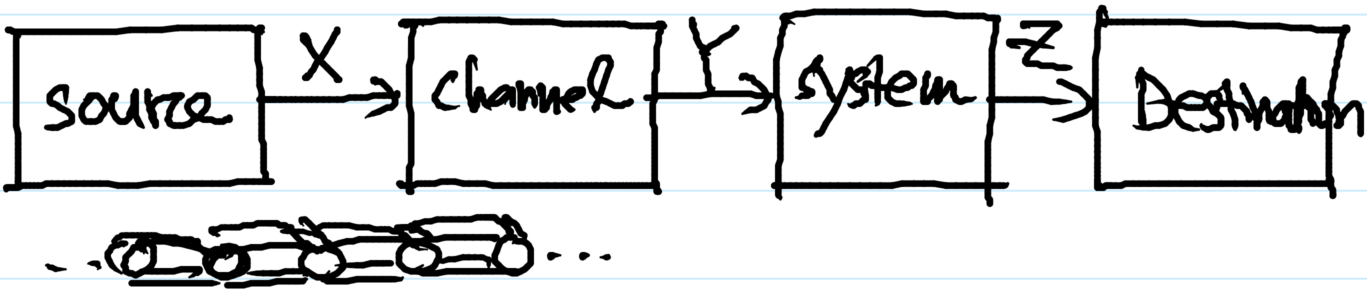
- Data Processing Theorem



$$I(X;Y) \geq I(X;Z)$$

- Markov chain

Def. Given 3 jointly distributed r.v.'s X, Y, and Z, we say X, Y, and Z form a Markov chain if

$$P_{X,Y,Z}(x,y,z) = P_X(x) P_{Y|X}(y|x) P_{Z|Y}(z|y)$$

$\forall x, \forall y, \forall z.$

$P_{Z|X,Y}(z|x,y)$

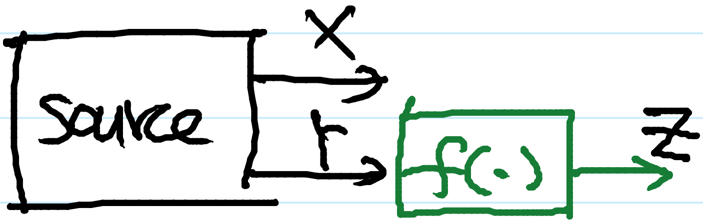
($\Leftrightarrow$ X & Z are conditionally independent given Y).

- Notation $X \to Y \to Z$

$\equiv Z \to Y \to X$

- Properties

(iii) Given two jointly distributed discrete r.v.'s X and Y, let $Z = f(Y)$ w/ f a deterministic function.



$$X \to Y \to Z$$

(iv) $X \to Y \to Z \Rightarrow I(X;Y) \geq I(X;Z)$

$\Updownarrow \qquad \Rightarrow I(Z;Y) \geq I(Z;X)$

$I(X;Z|Y) = 0$

Proof)

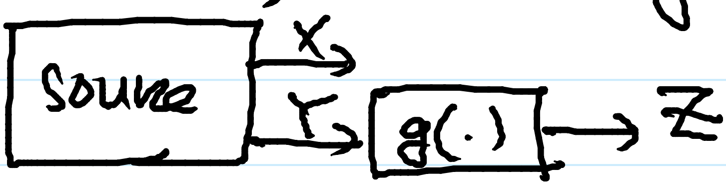
$$I(X;Y,Z) = I(X;Y) + \underbrace{I(X;Z|Y)}_{0}$$

$$= I(X;Z) + \underbrace{\boxed{I(X;Y|Z)}}_{\geq 0}$$

$5 = x + y \qquad y \geq 0$

$5 \geq x$

(v) Given X & Y, let $Z = g(Y)$. Then, $I(X;Y) \geq I(X;g(Y))$



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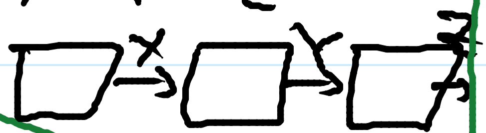
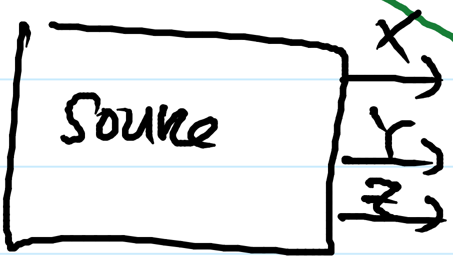
오후 1:00

(vi) Q. Given X, Y, Z, what is the correct inequality relation b/w

$$I(X;Y|Z) \boxed{\le} I(X;Y)$$

X→Y→Z





$$I(X;Y,Z) \ge I(X;Y)$$

(proof)

$$\underbrace{I(X;Z)}_{\ge 0} + I(X;Y|Z) = I(X;Y,Z) = I(X;Y)$$

$$\boxed{I(X;Y|Z) \le I(X;Y)}$$

where equality holds iff X & Z are indep

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오후 1:00

Data compression

- zero-error data compression
- fixed-length source coding
- Shannon's information metrics
- channel coding theorem
  - discrete channel
  - continuous channel (Gaussian)
  - continuous-time channel (〃)
- rate distortion theory

data 〃