Lec. 13

2013년 4월 10일 수요일

오후 8:56

Review

Shannon's information measures — all discrete r.v.'s.

- Entropy $H(X)$, $H(Y)$
- Joint Entropy $H(X,Y)$
- Conditional Entropy $H(X|Y)$, $H(Y|X)$
- Mutual Information $I(X;Y)$
- KL distance $D(P_X \| P_Y)$

Fano's inequality $H(X|\hat{X}) \le \cdots Pr(X \neq \hat{X})$

Information inequality $X \to Y \to Z$

$$\Rightarrow I(X;Z) \le I(X;Y)$$

Channel Coding Theorem

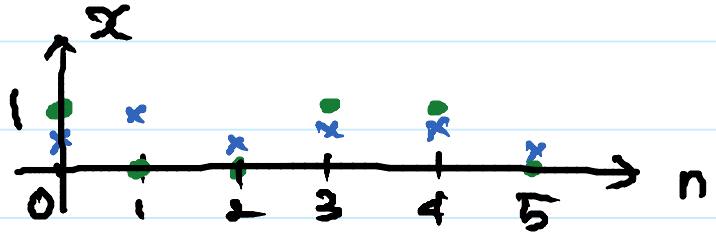
- DMC (Discrete Memoryless Channel)
  - $h(X)$, $h(X|Y)$, $h(X,Y)$, $I(X;Y)$, ...
- CMC (Continuous " ")

r.v: discrete, cont., mixed.

In particular, AWGN

DT

CT Bandlimited AWGN channel



Q. x is a discrete-time signal?

A. Yes.

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- Def. A system is a triplet $(\mathcal{D}, f, \mathcal{R})$

  where $\mathcal{D}$: domain

  $\mathcal{R}$: range

  f : deterministic mapping rule

- Def. A channel is a triplet $(\mathcal{X}, P, \mathcal{Y})$

  where $\mathcal{X}$: domain

  $\mathcal{Y}$: range

  P : random mapping rule

- Def. A discrete channel

  $\mathcal{X}$: a finite set

  $\mathcal{Y}$: a finite set

  $p(y|x)$: a conditional mass function

- Def. A DMC channel is a memoryless extension of a discrete channel $(\mathcal{X}, p(y|x), \mathcal{Y})$

  $\mathcal{X}^n$:

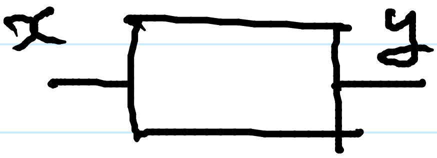
  $\mathcal{Y}^n$:

$$p(\underline{y}|\underline{x}) = \prod_{i=1}^{n} p(y_i|x_i)$$

$A = \{0,1\}$

$A^2 = \{(0,0),(0,1),(1,0),(1,1)\}$

example.



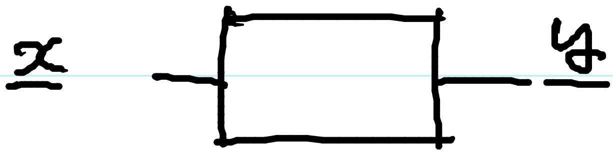
$\mathcal{X} = \{R, G\}$

$\mathcal{Y} = \{0, 1\}$

$p(0|R) = 1/4$

$p(0|G) = 1/3$

2nd-order memoryless extension.



$$\mathcal{X}^2 = \left\{ \begin{bmatrix} R \\ R \end{bmatrix}, \begin{bmatrix} R \\ G \end{bmatrix}, \begin{bmatrix} G \\ R \end{bmatrix}, \begin{bmatrix} G \\ G \end{bmatrix} \right\}$$

$$\mathcal{Y}^2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

$$p\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} \middle| \begin{bmatrix} R \\ G \end{bmatrix}\right) = p(0|R)\,p(0|G) = 1/4 \cdot 1/3 = 1/12$$

- Channel Coding Problem

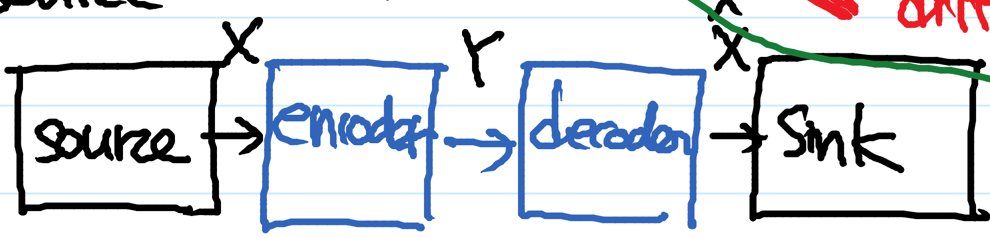
$\updownarrow$

Source " "

Find/Search for what you already know.

commonalities

differences



(i) compression rate

(ii) error rate

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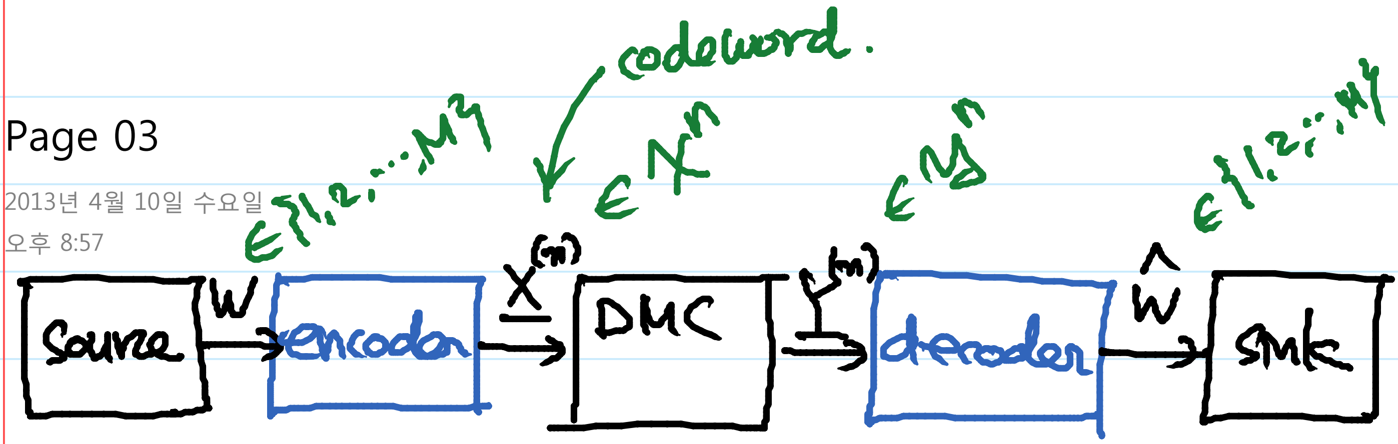

(i) transmission rate

(ii) error rate

$Pr(W=w_1)$,

$Pr(W=w_2)$,

$\vdots$

$Pr(W=w_M)$

(M,n) code

$$R \triangleq \frac{\log_2 M}{n} \quad [\text{bits}/\text{channel use}]$$

$Pr(W=w_1)$

$\vdots$

$Pr(W=w_{2^{nR}})$

$$\frac{\log_2 2^{nR}}{n} = R$$

$(\lceil 2^{nR} \rceil, n)$ code