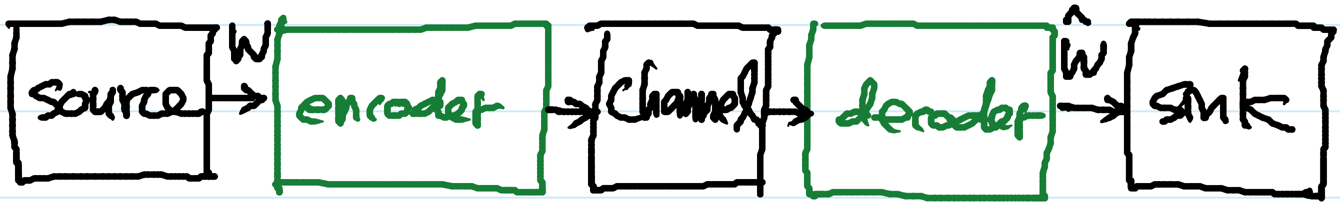
Lec. 14

2013년 4월 11일 목요일

오후 1:54

- Review

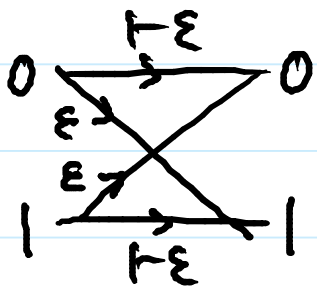
Channel Coding Problem



(i) transmission rate [bits/channel use]

(ii) error rate

ex)



Def. A transmission R is achievable if there exists a sequence of channel codes $\mathcal{C}^{(k)}$ such that

(i) $R^{(k)} \to R$ and

(ii) $P_e^{(k)} \to 0$

Q1 Given a channel, what is $\underline{\sup R}$?

$C_o \triangleq$ operational channel capacity

Q2. How to achieve it?

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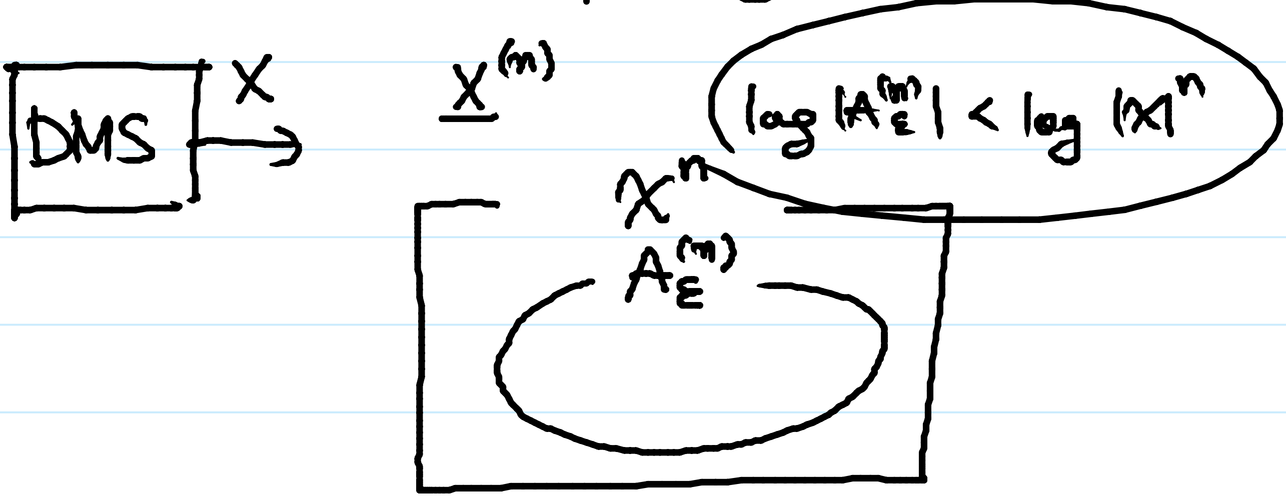
- Review of AEP (asymptotic equi-partition property)



$$A_\varepsilon^{(n)} = \left\{ (x_1, \cdots, x_n) \in \mathcal{X}^n : \left| \frac{1}{n} \sum_{i=1}^{n} \log \frac{1}{P_X(x_i)} - H(X) \right| < \varepsilon \right\}$$

↑ ε-typical set

(i) $P(\underline{X}^{(n)} \in A_\varepsilon^{(n)}) > 1-\varepsilon$ for sufficiently large n,

(ii) If $\underline{x}^{(n)} \in A_\varepsilon^{(n)}$, then

$$2^{-n(H(X)+\varepsilon)} < \Pr(\underline{X}^{(n)} = \underline{x}^{(n)}) < 2^{-n(H(X)-\varepsilon)}$$

(iii) $|A_\varepsilon^{(n)}| < 2^{n(H(X)+\varepsilon)}$

(iv) $|A_\varepsilon^{(n)}| > (1-\varepsilon)\, 2^{n(H(X)-\varepsilon)}$ for s.l. n

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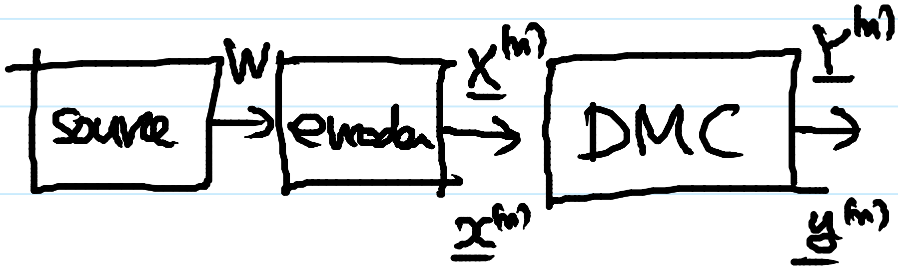
$$x \to \boxed{DC} \to y$$

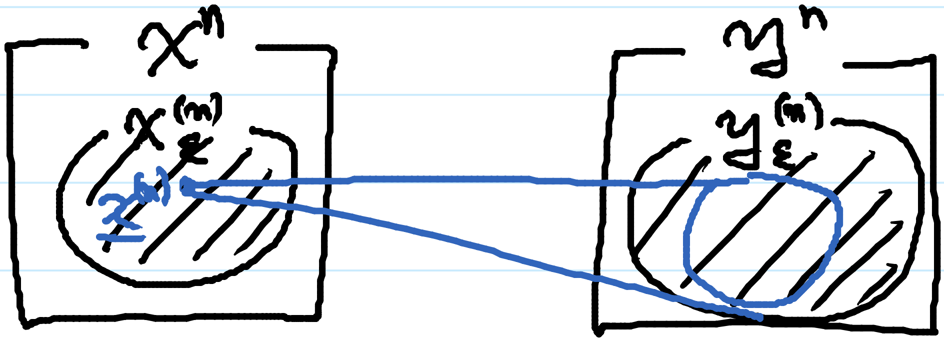
- DMC : $(\mathcal{X}, p(y|x), \mathcal{Y})$

$$\left(\mathcal{X}^n, \prod_{i=1}^{n} p(y_i|x_i), \mathcal{Y}^n\right)$$



Q. $P_X(x)$ & $P_{Y|X}(y|x)$, can you find $P_Y(y)$? (from $p_Y(y)$)
Given

- Choose $P_X(x)$, & ε
Generate $\underline{X}^{(n)}$ & $\underline{Y}^{(n)}$
Then,



If $\underline{x}^{(n)} \in \mathcal{X}_\varepsilon^{(n)}$, then $\left|\frac{1}{n}\sum_{i=1}^{n}\log\frac{1}{P_X(x_i)} - \sum_{i=1}^{|\mathcal{X}|} p_i \log\frac{1}{p_i}\right| < \varepsilon$

$P_X(0) = 1/4$
$P_X(1) = 1/4$
$P_X(2) = 1/2$

then $\underline{y}^{(n)}$ follows $\prod_{i=1}^{n} P(y_i|x_i)$

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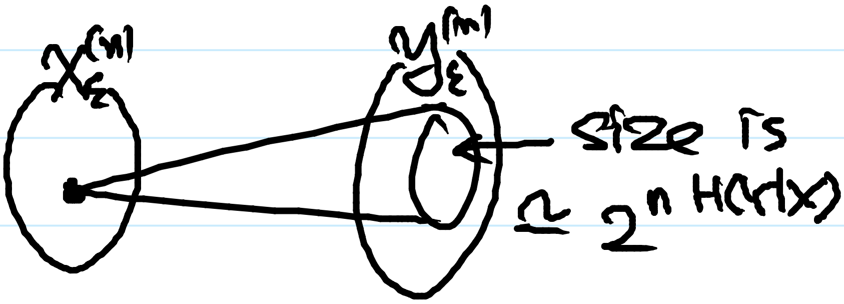
In other words, if $\underline{x}^{(n)} \in \mathcal{X}_\varepsilon^{(n)}$, then

$$-\frac{1}{n}\sum_{i=1}^{n}\log P_{Y|X}(y_i|x_i) - \underbrace{H(Y|X)}_{} \simeq 0$$

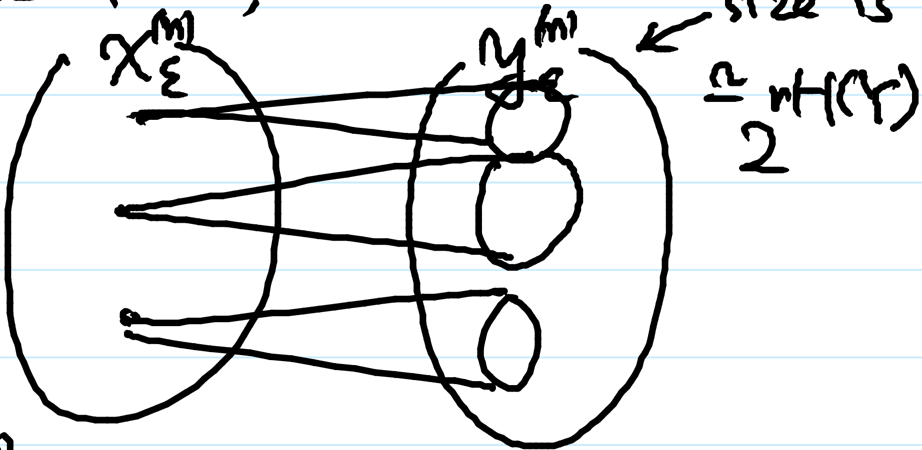
$$\sum_{x\in\mathcal{X}} H(Y|X=x)\Pr(X=x)$$

$$= \sum_{x\in\mathcal{X}}\left(\sum_{y\in\mathcal{Y}} P_{Y|X=x}(y)\log\frac{1}{P_{Y|X=x}(y)}\right)\Pr(X=x)$$

Consequently,



Too good to be true,



sphere packing argument!

$$\frac{2^{nH(Y)}}{2^{nH(Y|X)}} = 2^{n(H(Y)-H(Y|X))}$$

$$= 2^{nI(X;Y)}$$

$$\frac{\log 2^{nI(X;Y)}}{} = \frac{nI(X;Y)}{} = I(X;Y)$$

$$\frac{\log 2^{nI(X;Y)}}{n} = \frac{nI(X;Y)}{n} = I(X;Y)$$

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$$C = \max_{p_X(x)} I(X;Y)$$

informational channel capacity

- Channel Coding Theorem says,
  $\max_{p_X(x)} I(X;Y)$ is the operational channel capacity.

  $\Rightarrow$ direct part. achievability
  $\Leftarrow$ converse