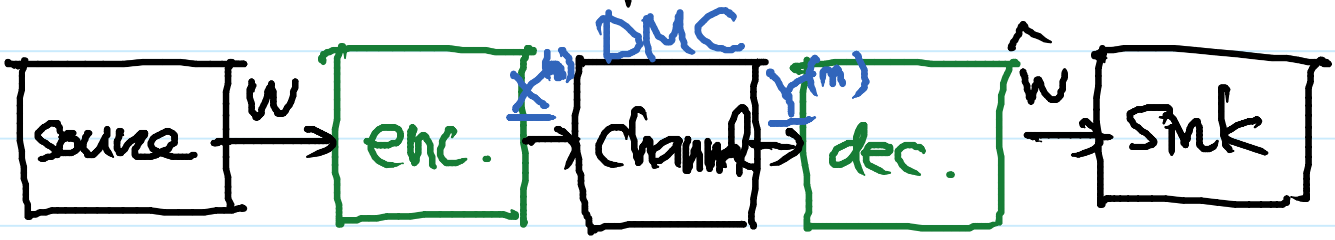
Lec. 15

2013년 4월 16일 화요일

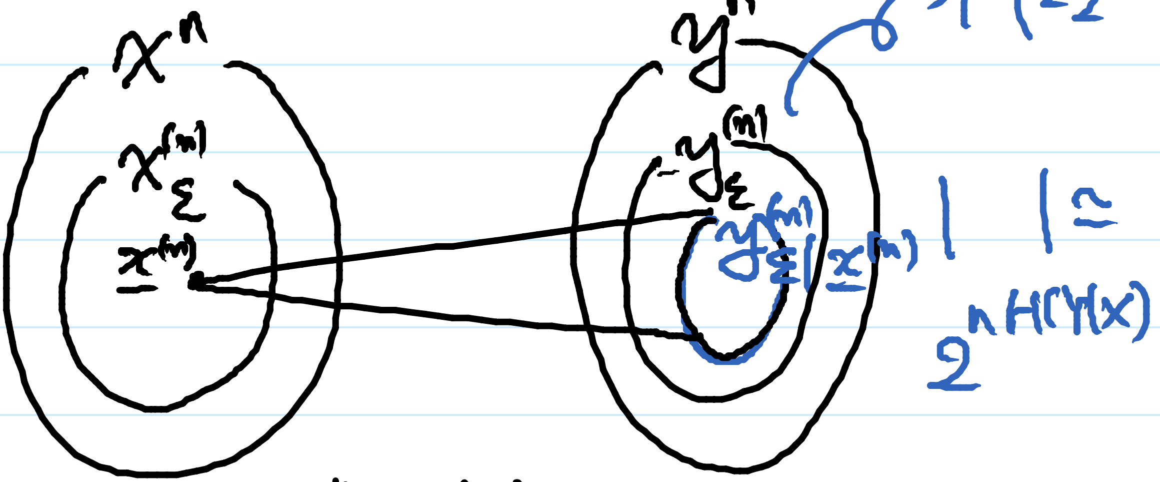
오후 1:54

## Review

- A sketch of the proof of NCCT.




- Joint AEP?




$P_X(x)$, $\varepsilon$ $\qquad\qquad$ $P_{Y|X}(y|x)$

| | | |
|---|---|---|
| 1 | 1/4 | $P_{Y|X}(y|1)$ |
| 2 | 1/4 | $P_{Y|X}(y|2)$ |
| 3 | 1/2 | $P_{Y|X}(y|3)$ |

$$\sum_{x\in\mathcal{X}} H(Y|X=x)\Pr(X=x) = H(Y|X)$$

$$\prod_{i=1}^{n} P_{Y|X}(y_i|x_i)$$

$$\frac{1}{n}\log\frac{1}{\prod_{i=1}^{n} P_{Y|X}(y_i|x_i)}$$

$$\frac{2^{nH(Y)}}{2^{nH(Y|X)}} = 2^{nI(X;Y)}$$

$$C = \max I(X;Y)$$

$$= -\frac{1}{n}\sum \log P_{Y|X}(y_i|x_i) \approx H(Y|X)$$

$p_X(x)$

- To prove,

  $$R \le \max_{P_X(x)} I(X;Y) \Longleftrightarrow R \text{ is achievable}$$

  $\equiv \Big($ $\Rightarrow$ Direct part. Achievability — Joint AEP

  $\quad\;\;$ $\Leftarrow$ Converse — Fano's inequality

  $$\equiv \; R > \max_{P_X(x)} I(X;Y) \Rightarrow R \text{ not achievable.}$$

- An M-ary message W.
  n times channel use.

| enc | |
|---|---|
| $w$ | $x^{(n)}$ |
| 1 | $c_1^{(n)}$ |
| 2 | $c_2^{(n)}$ |
| ⋮ | ⋮ |
| M | $c_M^{(n)}$ |

an $(M,n)$ code

$\Downarrow$

$$M = \lceil 2^{nR} \rceil$$

$$k \to \infty \Rightarrow \begin{cases} n_k \uparrow \infty \\ R_k \to R \\ P_{e,k} \to 0 \end{cases}$$

an $(\lceil 2^{n_k R} \rceil, n_k)$ code

- W is an M-ary message
  W is represented by $\underline{b}$ where
  the length is $\lceil \log_2 M \rceil = n$

$$0 \le \frac{\sum 1_{b_i \ne \hat{b}_i}}{n} \le \Pr(W \ne \hat{W}) \le \lambda$$

no problem!

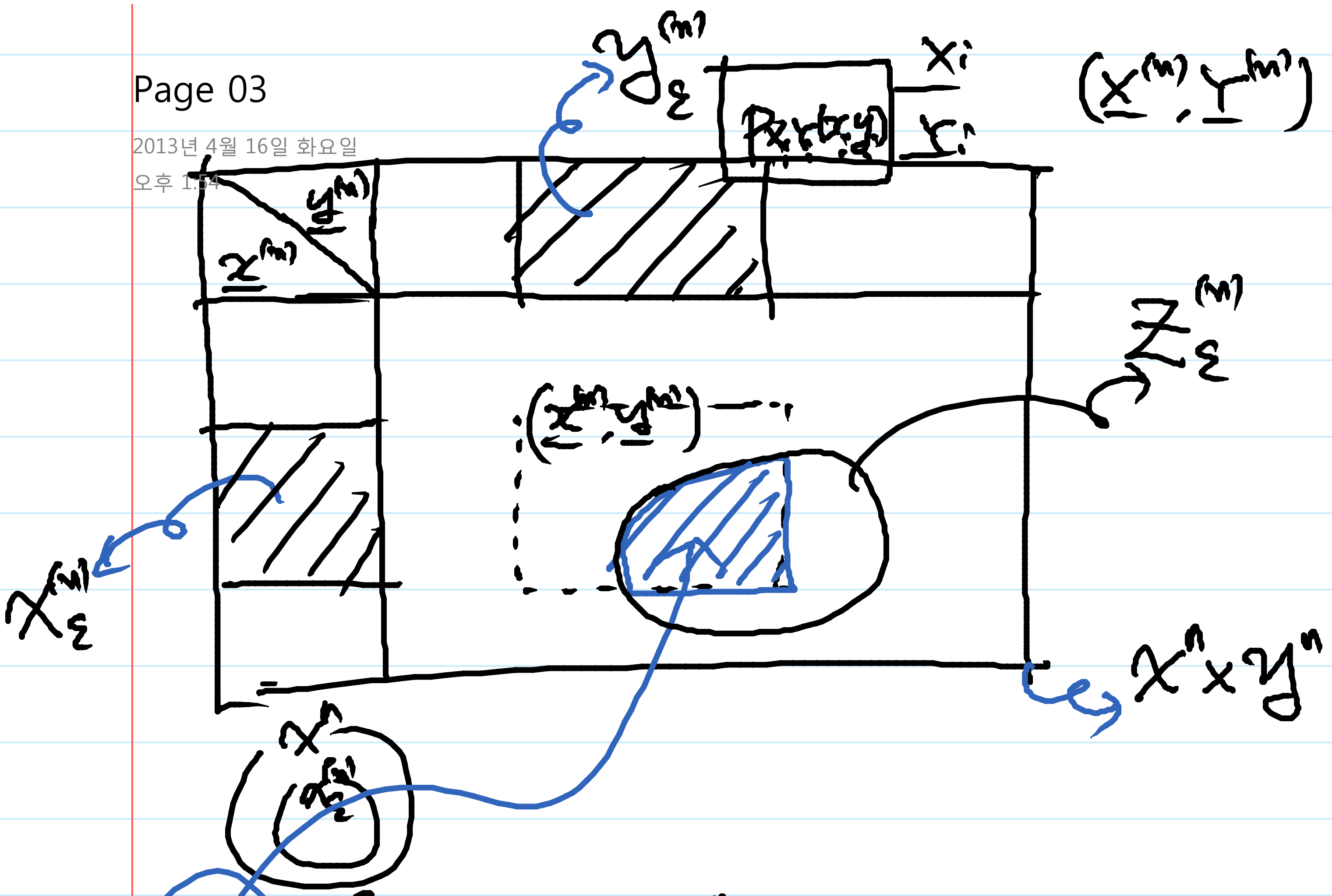
To prove achievability, we use $\Pr(W \ne \hat{W})$
To prove converse, as also " "
→ weak converse.

- Given a sequence of $(\lceil 2^{n_k R} \rceil, n_k)$ codes,
  the code rate is

$x \le \lceil x \rceil < x+1$

$$\frac{\log_2 2^{n_k R}}{n_k} \le \frac{\log_2 \lceil 2^{n_k R} \rceil}{n_k} < \frac{\log_2 (2^{n_k R} + 1)}{n_k}$$

$$R \le \lim_{n_k \to \infty} \frac{\log_2 \lceil 2^{n_k R} \rceil}{n_k} \le R$$




$A_\varepsilon^{(n)} \triangleq \{(\underline{x}^{(n)}, \underline{y}^{(n)}) \in \mathcal{X}^n \times \mathcal{Y}^n :$

(i) $\left|-\frac{1}{n}\log P_{\underline{X}^{(n)}}(\underline{x}^{(n)}) - H(X)\right| < \varepsilon,$

(ii) $\left|-\frac{1}{n}\log P_{\underline{Y}^{(n)}}(\underline{y}^{(n)}) - H(Y)\right| < \varepsilon,$

and (iii) $\left|-\frac{1}{n}\log P_{\underline{X}^{(n)},\underline{Y}^{(n)}}(\underline{x}^{(n)},\underline{y}^{(n)}) - H(X,Y)\right| < \varepsilon\}$

- $|A_\varepsilon^{(n)}|$

$(\underline{X}^{(n)}, \underline{Y}^{(n)}) \sim \prod_{i=1}^{n} P_{X,Y}(x_i, y_i)$

$\Pr((\underline{X}^{(n)}, \underline{Y}^{(n)}) \in A_\varepsilon^{(n)})$

$\Pr((\tilde{\underline{X}}^{(n)}, \tilde{\underline{Y}}^{(n)}) \in A_\varepsilon^{(n)})$

$\uparrow$ $P_X(x)$ $\quad$ $\uparrow$ $P_Y(y)$

$(\tilde{\underline{X}}^{(n)}, \tilde{\underline{Y}}^{(n)}) \sim \prod_{i=1}^{n} P_X(x_i) P_Y(y_i)$

$\uparrow \quad \uparrow \qquad (\underline{X}, \underline{Y}) \sim \prod_{i=1} p_X(x_i) p_Y(y_i)$

$p_X(x) \quad p_Y(y)$