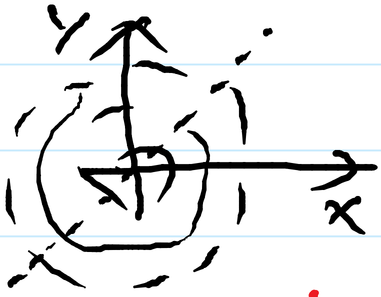
Lec. 16

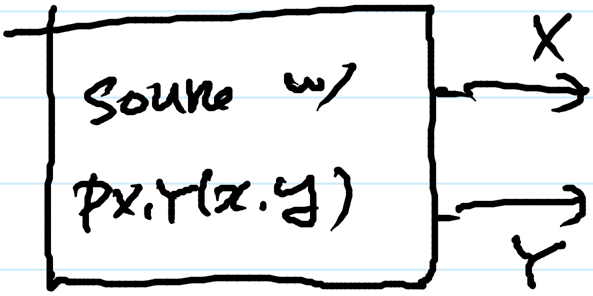
2013년 4월 18일 목요일

오후 1:56

Review

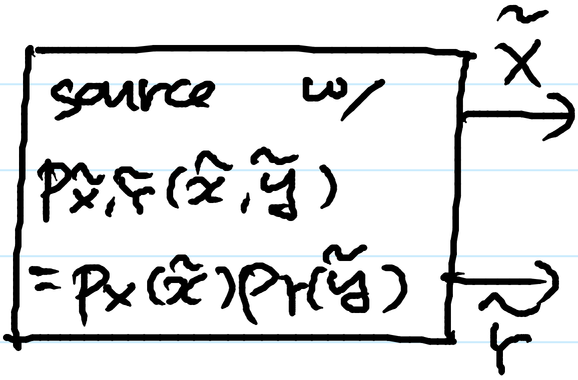
Joint AEP





$(X^{(n)}, Y^{(n)})$

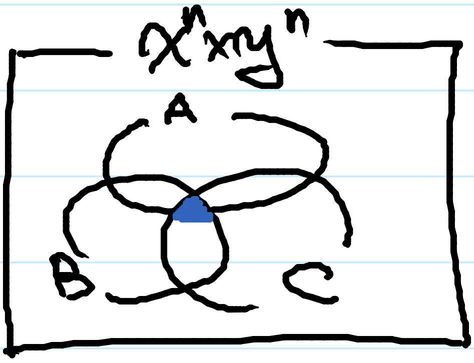
abstract
↕
concrete



$(\tilde{X}^{(n)}, \tilde{Y}^{(n)})$

$$A_\varepsilon^{(n)} \triangleq \Big\{ (x^{(n)}, y^{(n)}) \in \mathcal{X}^n \times \mathcal{Y}^n : \\ \Big| -\frac{1}{n}\sum_{i=1}^{n} \log p_X(x_i) - H(X) \Big| < \varepsilon, \\ \Big| -\frac{1}{n}\sum_{i=1}^{n} \log p_Y(y_i) - H(Y) \Big| < \varepsilon, \\ \Big| -\frac{1}{n}\sum_{i=1}^{n} \log p_{X,Y}(x_i, y_i) - H(X,Y) \Big| < \varepsilon \Big\}$$

$$= A \cap B \cap C$$



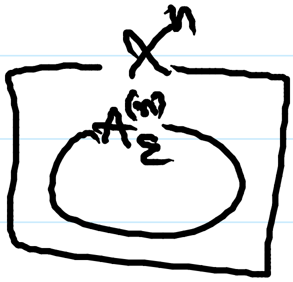
intuitive / un-intuitive / counter- " → trivial / non-trivial ⇒ implication, application.

(i) $|A_\varepsilon^{(n)}|$?

(ii) $\Pr\{(\underline{X}^{(n)}, \underline{Y}^{(n)}) \in A_\varepsilon^{(n)}\}$?

(iii) $\Pr\{(\tilde{\underline{X}}^{(n)}, \tilde{\underline{Y}}^{(n)}) \in A_\varepsilon^{(n)}\}$?

- Review of AEP



$$1 \ge \sum_{\underline{x}^{(n)} \in A_\varepsilon^{(n)}} \Pr(\underline{X}^{(n)} = \underline{x}^{(n)}) \ge \cdots$$

$$2^{-n(H(X)+\varepsilon)} \le P_X(\underline{X}^{(n)} = \underline{x}^{(n)}) \le 2^{-n(H(X)-\varepsilon)}$$

- (i)

$$1 \ge \sum_{(\underline{x}^{(n)}, \underline{y}^{(n)}) \in A_\varepsilon^{(n)}} \Pr((\underline{X}^{(n)}, \underline{Y}^{(n)}) = (\underline{x}^{(n)}, \underline{y}^{(n)}))$$

$$\ge \sum 2^{-n(H(X,Y)+\varepsilon)}$$

$$= 2^{-n(H(X,Y)+\varepsilon)} \, |A_\varepsilon^{(n)}|$$

$$\Rightarrow \quad |A_\varepsilon^{(n)}| \le 2^{n(H(X,Y)+\varepsilon)}$$

(i)'

$$1-\varepsilon \le \sum_{(\underline{x}^{(n)}, \underline{y}^{(n)}) \in A_\varepsilon^{(n)}} \Pr((\underline{X}^{(n)}, \underline{Y}^{(n)}) = (\underline{x}^{(n)}, \underline{y}^{(n)}))$$

f.s.l. n.

$$\le |A_\varepsilon^{(n)}| \, 2^{-n(H(X,Y)-\varepsilon)}$$

2013년 4월 18일 목요일

오후 1:56

$$(i),(i)' \Rightarrow (1-\varepsilon)2^{n(H(X,Y)-\varepsilon)} < |A_\varepsilon^{(n)}| < 2^{n(H(X,Y)+\varepsilon)}$$

for s.l. n

$$(ii)\ \Pr\left((\underline{X}^{(n)},\underline{Y}^{(n)}) \in A_\varepsilon^{(n)} \right)$$

$$= \Pr\left(\underline{X}^{(n)} \in \mathcal{X}_\varepsilon^{(n)} \text{ and } \underline{Y}^{(n)} \in \mathcal{Y}_\varepsilon^{(n)} \text{ and } (\underline{X}^{(n)},\underline{Y}^{(n)}) \in (\mathcal{X}^n \times \mathcal{Y}^n)_\varepsilon \right)$$

$$\Pr\left((\underline{X}^{(n)},\underline{Y}^{(n)}) \notin A_\varepsilon^{(n)} \right) = 1 - \Pr\left((\underline{X}^{(n)},\underline{Y}^{(n)}) \in A_\varepsilon^{(n)} \right)$$

$$= \Pr\left(\underline{X}^{(n)} \notin \mathcal{X}^{(n)} \text{ or } \underline{Y}^{(n)} \notin \mathcal{Y}^{(n)} \text{ or } (\underline{X}^{(n)},\underline{Y}^{(n)}) \notin (\mathcal{X}^n \times \mathcal{Y}^n)_\varepsilon \right)$$

Union bound technique

$$\le \Pr(\underline{X}^{(n)} \notin \mathcal{X}^{(n)}) + \Pr(\underline{Y}^{(n)} \notin \mathcal{Y}^{(n)}) + \Pr((\underline{X}^{(n)},\underline{Y}^{(n)}) \notin (\mathcal{X}^n \times \mathcal{Y}^n)_\varepsilon)$$

$$< \varepsilon + \varepsilon + \varepsilon \quad \text{f.s.l. } n.$$

$$= \varepsilon'$$

$$\Rightarrow \Pr\left((\underline{X}^{(n)},\underline{Y}^{(n)}) \in A_\varepsilon^{(n)} \right) > 1-\varepsilon' \quad \text{for s.l. n}$$

$$(iii)\ \Pr\left((\underline{\tilde{X}}^{(n)},\underline{\tilde{Y}}^{(n)}) \in A_\varepsilon^{(n)} \right)$$

Intelligence "exhaustive search"

$$= \sum_{(\underline{\tilde{x}}^{(n)},\underline{\tilde{y}}^{(n)}) \in A_\varepsilon^{(n)}} \Pr\left((\underline{\tilde{X}}^{(n)},\underline{\tilde{Y}}^{(n)}) = (\underline{\tilde{x}}^{(n)},\underline{\tilde{y}}^{(n)}) \right)$$

2013년 4월 18일 목요일

오후 1:57

$$= \sum_{(\tilde{x}^{(n)}, \tilde{y}^{(n)}) \in A_\epsilon^{(n)}} \underbrace{\Pr(\tilde{X}^{(n)} = \tilde{x}^{(n)})}_{\simeq 2^{-nH(X)}} \underbrace{\Pr(\tilde{Y}^{(n)} = \tilde{y}^{(n)})}_{\simeq 2^{-nH(Y)}}$$

$$(1-\epsilon)\, 2^{n(H(X,Y)-\epsilon)} < |A_\epsilon^{(n)}| < 2^{n(H(X,Y)+\epsilon)}$$

f.s.l.n

$$\le 2^{-n(H(X)-\epsilon)}\, 2^{-n(H(Y)-\epsilon)}\, 2^{n(H(X,Y)+\epsilon)}$$

$$= 2^{-n(H(X)+H(Y)-H(X,Y)-3\epsilon)}$$

$$= 2^{-n(I(X;Y)-3\epsilon)} \quad \text{f.s.l.n}$$

* $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$

$\sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$

rate of convergence

- The idea of random coding

simplistic

Source w/ infinite # of bits $\rightarrow$ $\Rightarrow$ random source w/ $\Pr(W=m_i)$ $\rightarrow$

Page 04

2013년 4월 18일 목요일

오후 1:57

$$\boxed{\text{source}} \xrightarrow{W} \boxed{\text{enc.}} \rightarrow \boxed{\text{ch.}} \rightarrow \boxed{\text{dec.}} \xrightarrow{\hat{W}} \boxed{\text{snk}}$$

$P_X(x)$

m_i	$x_i^{(n)}$
1	$\underline{s}_1^{(n)}$
2	$\vdots$
$\vdots$	$\vdots$
$\lceil 2^{nR} \rceil$	$\underline{s}_{\lceil 2^{nR} \rceil}^{(n)}$

$\mathcal{C}_1 \qquad \mathcal{C}_2 \quad \cdots\cdots$

When,

$0 \le x_i$, if $\frac{1}{n}\sum_{i=1}^{n} x_i < a$.

then $\exists x_j$ s.t. $x_j < a$.

한놈!

$0 \le y_i$, if $\frac{1}{m}\sum_{i=1}^{m} y_i < b$.

여러놈!

then $y_1 \le y_2 \le \cdots \le y_{\frac{m}{2}} \le 2b$.

Let $y_1 \le y_2 \le \cdots \le y_m$

The Markov inequality.

$0 \le Y$, $\Pr(Y > 2\bar{Y}) \le \frac{1}{2}$. $P(Y \le 2\bar{Y}) \ge \frac{1}{2}$