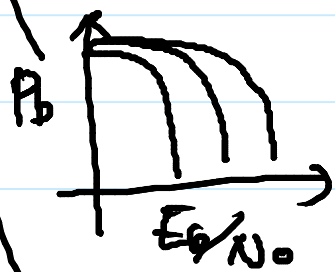
Lec.17

2013년 4월 23일 화요일

오후 1:54

Review

- data compression
  - zero-error v.l.s.c
  - a. small error b. coding
  - (lossy compression)
- " transmission
  - DMC
  - (DT AWGN)
  - (CT AWGN)



- KL distance & detection problem.

- v.l.s. coding
- source coding theorem
- Shannon's information measures
- channel coding theorem < DMC, AWGN
- rate distortion theorem

References

1. Cover & Thomas, Element of IT → Typical set decoding
2. Yeung, → discrete
3. Gallager, → ML decoding
4. El Gamal & Kim, NIT

common (2, 3, 4)

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- Channel Coding Problem

Q. Find $\sup R$ s.t. there exists a $(\lceil 2^{nR} \rceil, n)$ code having $\Pr(\hat{W} \neq W ; W = m_i) \simeq 0, \forall i$

(ex)

m_i	$\underline{c}_i$
$00 \cdots 0$	$\underline{x}_1$
$00 \cdots 1$	$\underline{x}_2$
$\vdots$	$\vdots$
$\underbrace{11 \cdots 1}_{100 \text{ bits}}$	$\underline{x}_{2^{100}}$

A. $$\sup R = \max_{p_X(x)} I(X;Y)$$

- Today, we prove the achievability.
  - (i) We employ the typical set decoder, seemingly, a suboptimal decoder. (Actually, it is asymptotically optimal.)
  - (ii) We employ the random $(\lceil 2^{nR} \rceil, n)$ code by using $p_X(x)$.
  - (iii) ← heights.

coins.

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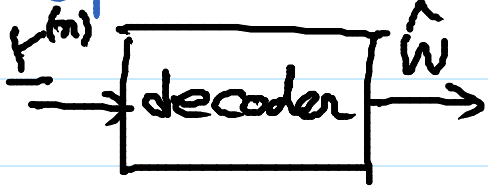
Typical Set Decoder

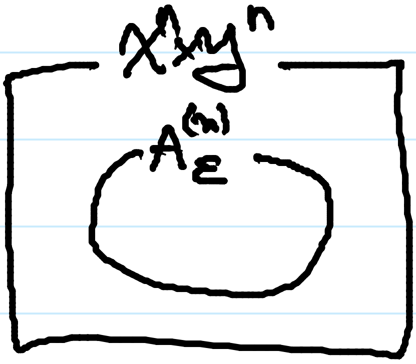


- The decoder observes $\{\underline{Y}^{(n)} = \underline{y}^{(n)}\}$.

$P_X(x)\,P_{Y|X}(y|x)$

$\hat{W} = m_i$ if (i) $(\underline{x}_i^{(n)}, \underline{y}^{(n)}) \in A_\varepsilon^{(n)}$

the codeword assigned to the i-th message



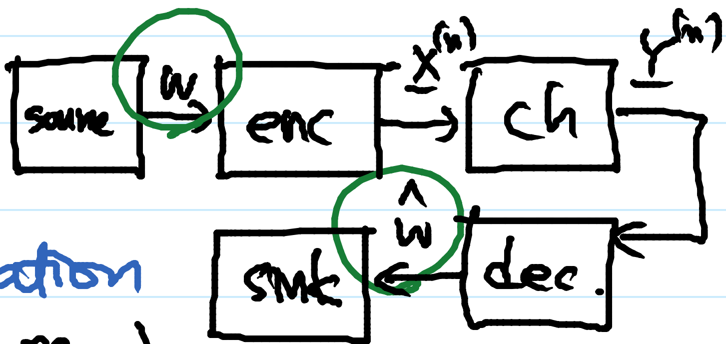
(ii) $(\underline{x}_j^{(n)}, \underline{y}^{(n)}) \notin A_\varepsilon^{(n)}, \ \forall j \neq i$.

$\hat{W}$ = arbitrary & declare error, otherwise.

(i)$'$ $(\underline{x}_j^{(n)}, \underline{y}^{(n)}) \notin A_\varepsilon^{(n)}, \ \forall j$.

(ii)$'$ more than one codeword is jointly typical.

- nothing
- only 1
- more than 1.



Performance Evaluation

$$\Pr(\hat{W} \neq m_1 \,;\, W = m_1)$$

$$= \sum_{\underline{x}^{(n)} \in X^n} \Pr(\hat{W} \neq m_1, \underline{X}_1^{(n)} = \underline{x}^{(n)} \,;\, W = m_1)$$

$$= \sum_{\underline{x}^{(n)} \in \mathcal{X}^n} Pr\left(\hat{W} \neq m_1 \mid \underline{X}_1^{(n)} = \underline{x}^{(n)} ; W = m_1\right) \times Pr\left(\underline{X}_1^{(n)} = \underline{x}^{(n)} \cancel{; W = m_1}\right)$$

Since we use the typical set decoder, $\{\hat{W} = m_1\}$

$$= \left\{ (\underline{X}_1^{(n)}, \underline{Y}^{(n)}) \in A_\varepsilon^{(n)} \right\} \cap \left\{ (\underline{X}_2^{(n)}, \underline{Y}^{(n)}) \notin A_\varepsilon^{(n)} \right\} \cap \cdots \cap \left\{ (\underline{X}_{\lceil 2^{nR} \rceil}^{(n)}, \underline{Y}^{(n)}) \notin A_\varepsilon^{(n)} \right\}$$

$$= \sum_{\underline{x}^{(n)} \in \mathcal{X}^n} Pr\Big(\left\{ (\underline{X}_1^{(n)}, \underline{Y}^{(n)}) \notin A_\varepsilon^{(n)} \right\} \cup \left\{ (\underline{X}_2^{(n)}, \underline{Y}^{(n)}) \in A_\varepsilon^{(n)} \right\} \cdots \cup \left\{ (\underline{X}_{\lceil 2^{nR} \rceil}^{(n)}, \underline{Y}^{(n)}) \in A_\varepsilon^{(n)} \right\} \Big| \underline{X}_1^{(n)} = \underline{x}^{(n)} ; W = m_1 \Big) \times Pr\left(\underline{X}_1^{(n)} = \underline{x}^{(n)}\right)$$

$$\leq \sum_{\underline{x}^{(n)} \in \mathcal{X}^n} Pr\left((\underline{X}_1^{(n)}, \underline{Y}^{(n)}) \notin A_\varepsilon^{(n)} \mid \underline{X}_1^{(n)} = \underline{x}^{(n)} \cancel{; W = m_1} \right) \times Pr\left(\underline{X}_1^{(n)} = \underline{x}^{(n)}\right)$$

$$+ \sum_{j \neq 1} \sum_{\underline{x}^{(n)} \in \mathcal{X}^n} Pr\left((\underline{X}_j^{(n)}, \underline{Y}^{(n)}) \in A_\varepsilon^{(n)} \mid \underline{X}_1^{(n)} = \underline{x}^{(n)} ; W = m_1 \right) \times Pr\left(\underline{X}_j^{(n)} = \underline{x}^{(n)}\right)$$

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$p_{X,Y}(x,y)$

$$= \Pr\left((X^{(n)}, Y^{(n)}) \notin A_\varepsilon^{(n)} \right)$$

$$+ \sum_{j \neq 1} \Pr\left((\tilde{X}^{(n)}, \tilde{Y}^{(n)}) \in A_\varepsilon^{(n)} \right)$$

$p_X(x)\, p_Y(y)$

$$= \Pr\left((X^{(n)}, Y^{(n)}) \notin A_\varepsilon^{(n)} \right) + \left(\lceil 2^{nR} \rceil - 1 \right) \Pr\left((\tilde{X}^{(n)}, \tilde{Y}^{(n)}) \in A_\varepsilon^{(n)} \right)$$

$$\leqq \varepsilon + \left(\lceil 2^{nR} \rceil - 1 \right) 2^{-n(I(X;Y) - 3\varepsilon)}$$

cf) $x \le \lceil x \rceil \le x+1$

$$\le \varepsilon + 2^{nR}\, 2^{-n(I(X;Y) - 3\varepsilon)}$$

$$= \varepsilon + 2^{-n(I(X;Y) - R - 3\varepsilon)}$$

$$\therefore \Pr(\hat{W} = m_1 ; W = m_1) \to 0$$

if $I(X;Y) - R - 3\varepsilon > 0$

$$\left(\begin{array}{l} \Leftrightarrow \; R < I(X;Y) - 3\varepsilon \\ \Rightarrow \; R \le I(X;Y) \\ \Rightarrow \; R \le \max\limits_{p_X(x)} I(X;Y) \end{array} \right)$$