Lec. 20

2013년 4월 30일 화요일

오후 1:56

review

context

- **Review**

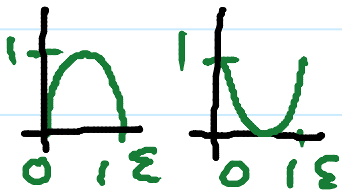
- Problem to solve : $\boxed{\begin{array}{l}\text{Given } P_{Y|X}(y|x) \\ \max\limits_{P_X(x)} I(X;Y)\end{array}}$

- $I(X;Y) = E\left\{\log \frac{P_{X,Y}(X,Y)}{P_X(X)\,P_Y(Y)}\right\}$
$= D(P_{X,Y} \,\|\, P_X P_Y)$

- Examples

(i) BSC $<$ cross-over prob. $\varepsilon = 0$; " " $\varepsilon \neq 0$.

$C = 1 - h_b(\varepsilon)$



(ii) NTC $\quad C = \log_2 13$
$= \log_2 26 - \log_2 2$

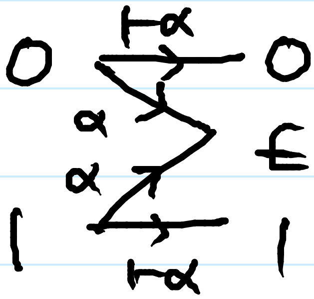
Symmetric ch.
* BSC
* BEC
* NTC

(iii) Symmetric

$C = \max I(X;Y) = H(Y) - H(Y|X)$
$= H(Y) - H(\underline{r})$
$\leq \log|\mathcal{Y}| - H(\underline{r})$

$C \leq \log|\mathcal{Y}| - H(\underline{r})$

(iii) BEC



$C \le \log_2 3 - h_b(\alpha)$

$$I(X;Y) = H(X) - H(X|Y)$$

$$= h_b(\beta) - \cancel{H(X|Y=0)\Pr(Y=0)} \rightarrow 0$$

$$- H(X|Y=E)\Pr(Y=E) \rightarrow \alpha$$

$$- \cancel{H(X|Y=1)\Pr(Y=1)}$$

$$h_b(\beta) \qquad = h_b(\beta)(1-\alpha)$$

$$P_{X|Y}(x|E) = \frac{\Pr(Y=E|X=x)\Pr(X=x)}{\Pr(Y=E)} = \frac{\cdots}{\alpha}$$

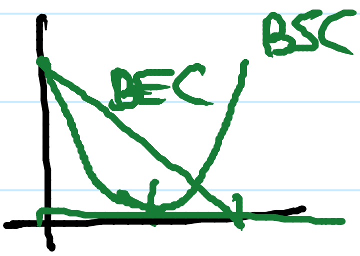
$$\Pr(Y=E) = \Pr(Y=E|X=0)\Pr(X=0) + \Pr(Y=E|X=1)\Pr(X=1)$$

$$= \alpha\beta + \alpha(1-\beta)$$

$$= \alpha(\beta + 1 - \beta)$$

$$= \alpha$$

$\therefore \quad C_{BEC} = 1-\alpha \quad$ [bits/ch.use]



2013년 4월 30일 화요일

오후 1:56

- Given $P_{Y|X}(y|x)$, the mutual information $I(X;Y)$ is a concave function of $p_X(x)$.

- Optimization Theory $\qquad \max_{x \in \Omega} f(x)$

f: single / multiple → objective o.p.

x: scalar, vector, matrix, / function ← calculus of variations.

Ω: discrete / continuous / mixed

convex optimization problems

⇒ "local optimal ⇒ global optimal"

⇒ "numerical search."

linear / nonlinear $\qquad$ cvx / non-cvx

Page 03

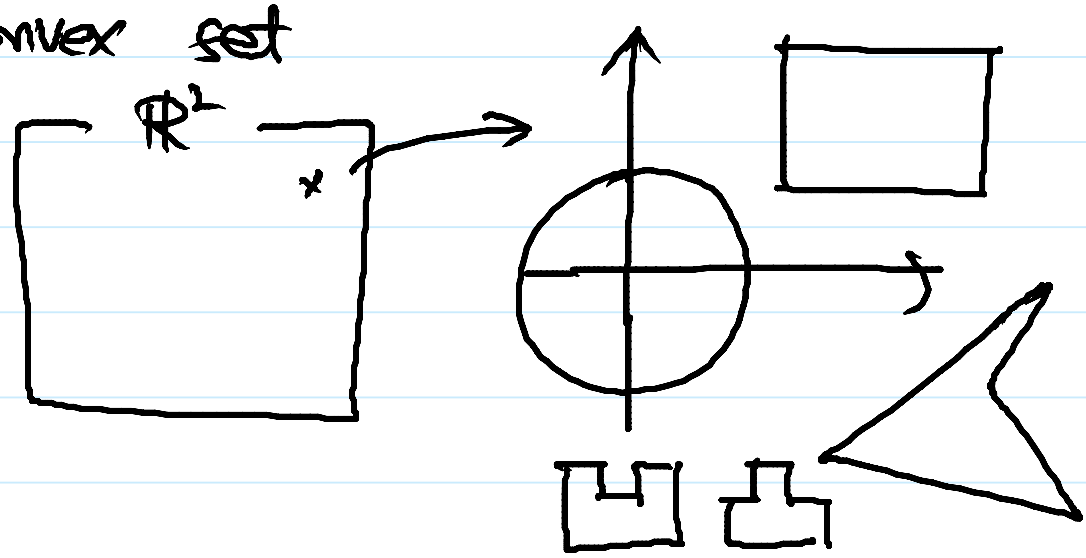
2013년 4월 30일 화요일

오후 1:56

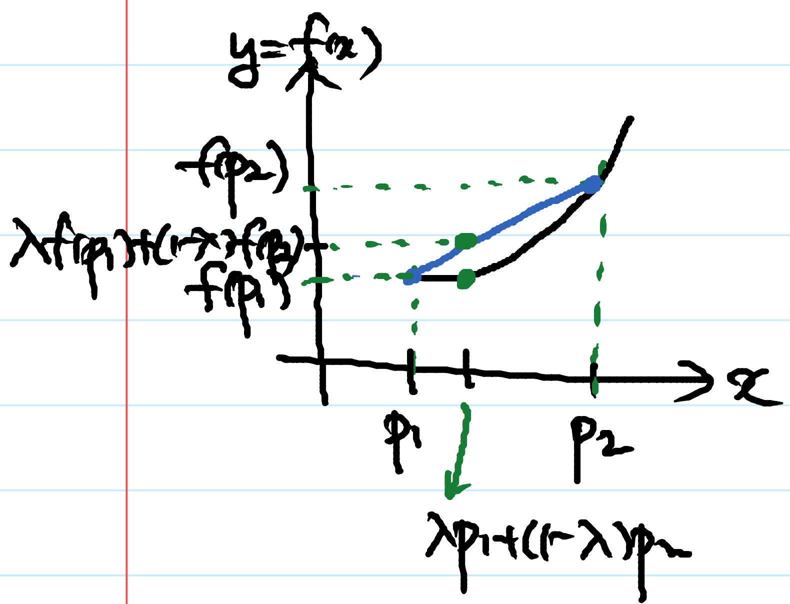
Stanford S. Boyd < CVX, YouTube

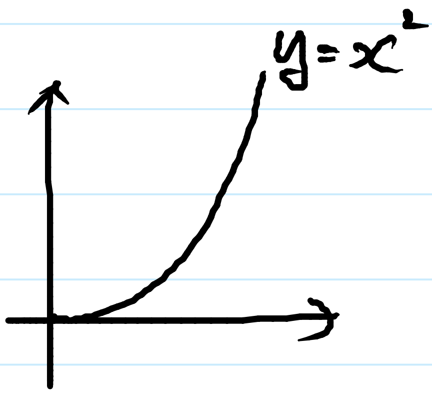
Princeton M. Chiang — CVX & comm.

• Convex set



• Convex function





• Convex optimization $\min_{x \in \Omega} f(x)$

- $f(\lambda p_1 + (1-\lambda)p_2) < \lambda f(p_1) + (1-\lambda) f(p_2)$, $\forall p_1 \neq p_2$

  strictly convex

- $\leq$ $\forall p_1, p_2 \in S$

  convex

- f is cvx $\Leftrightarrow$ $-f$ is concave.

Back to our problem:

- $\max_{p_X(x)} I(X;Y)$ Given X,

To show (i) the set of all $p_X(x)$ is a convex set

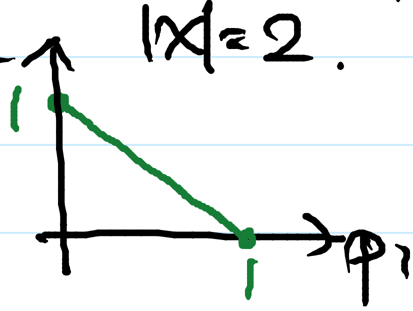
(ii) The function $I(X;Y)$ is a concave function of $p_X(x)$.

(i) Representation of $p_X(x)$: $\underline{p} \in$ length $|X|$

Example $|X| = 2$.

$0 \leq p_i \leq 1$

$p_1 + \cdots + p_{|X|} = 1$

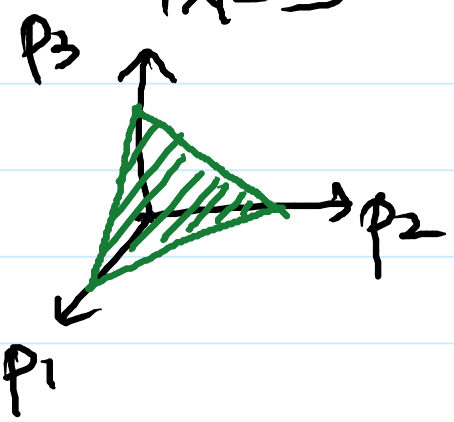


Page 05

2013년 4월 30일 화요일

오후 1:56

$|X| = 3$



(ii) p_1, p_2 $\quad \lambda p_1 + (1-\lambda) p_2 \triangleq p_\lambda$

To show

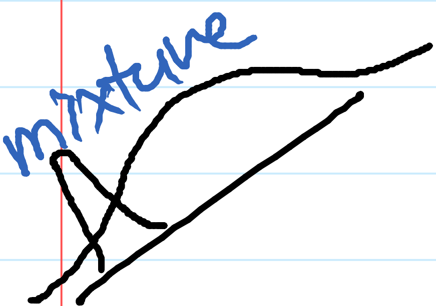
$$I(p_\lambda) \geq \lambda I(p_1) + (1-\lambda) I(p_2), \quad \forall p_1, p_2.$$

$I(p_1) = I(X;Y)$ evaluated w/ $p_1(x)\, p_{Y|X}(y|x)$

$I(p_2) = I(X;Y)$ evaluated w/ $p_2(x)\, p_{Y|X}(y|x)$

$I(p_\lambda) = I(X;Y)$ evaluated w/ $\{\lambda p_1(x) + (1-\lambda) p_2(x)\} \cdot p_{Y|X}(y|x)$

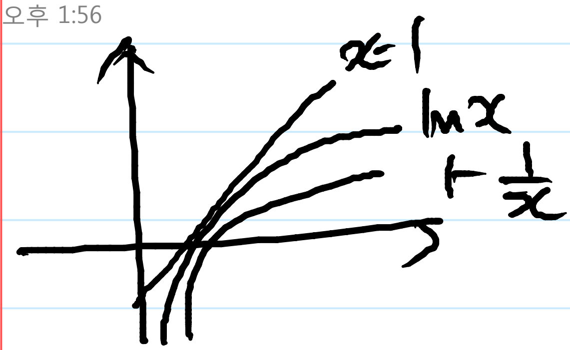
$= \lambda p_1(x) p_{Y|X}(y|x) + (1-\lambda) p_2(x) p_{Y|X}(y|x)$



Page 06

2013년 4월 30일 화요일

오후 1:56



$$P_{X|X}(y|x) = \begin{cases} 1, & x=y. \\ 0, & x\neq y. \end{cases}$$

$$H(X) = I(X;X)$$

CVX opt. problem: Existence. ~~Uniqueness~~. local $\Leftrightarrow$ global

$$C = \max_{P_X(x)} I(X;Y) = \max_{P_X(x)} H(X)-H(X|Y)$$

$$H(Y)-H(Y|X)$$

NTC.