Lec. 21

2013년 5월 1일 수요일

오후 8:56

Review

zero-error variable-length source coding

$\rightarrow H(X)$

arbitrarily small error fixed-length " "

$\rightarrow$ (AEP, typical set) $\rightarrow H(X)$

Shannon's information measures

$H(X)$, $H(X,Y)$, $H(X|Y)$, $I(X;Y)$,

$I(X;Y|Z)$, $D(p_{XY} \| p_X \cdot p_Y)$ ......

$(D(p_X \| p_Y) \geq 0)$

$H(X) \geq H(X|Y)$

$H(X,Y) \leq H(X)+H(Y)$

Fano's inequality : $H(X|\hat{X})$, $Pr(\hat{X} \neq X)$

Data Processing Inequality : Markov...

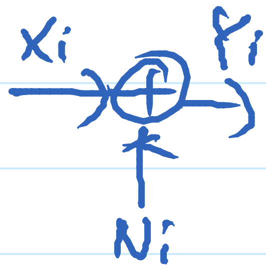
$h(X)$

continuous r.v.

Channel coding for a DMC

Sphere-Packing argument

$C = \max_{p_X(x)} I(X;Y)$



Achievability : Joint AEP typical set decoder...

Converse.

Symmetric channel.

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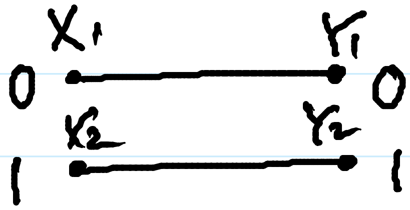
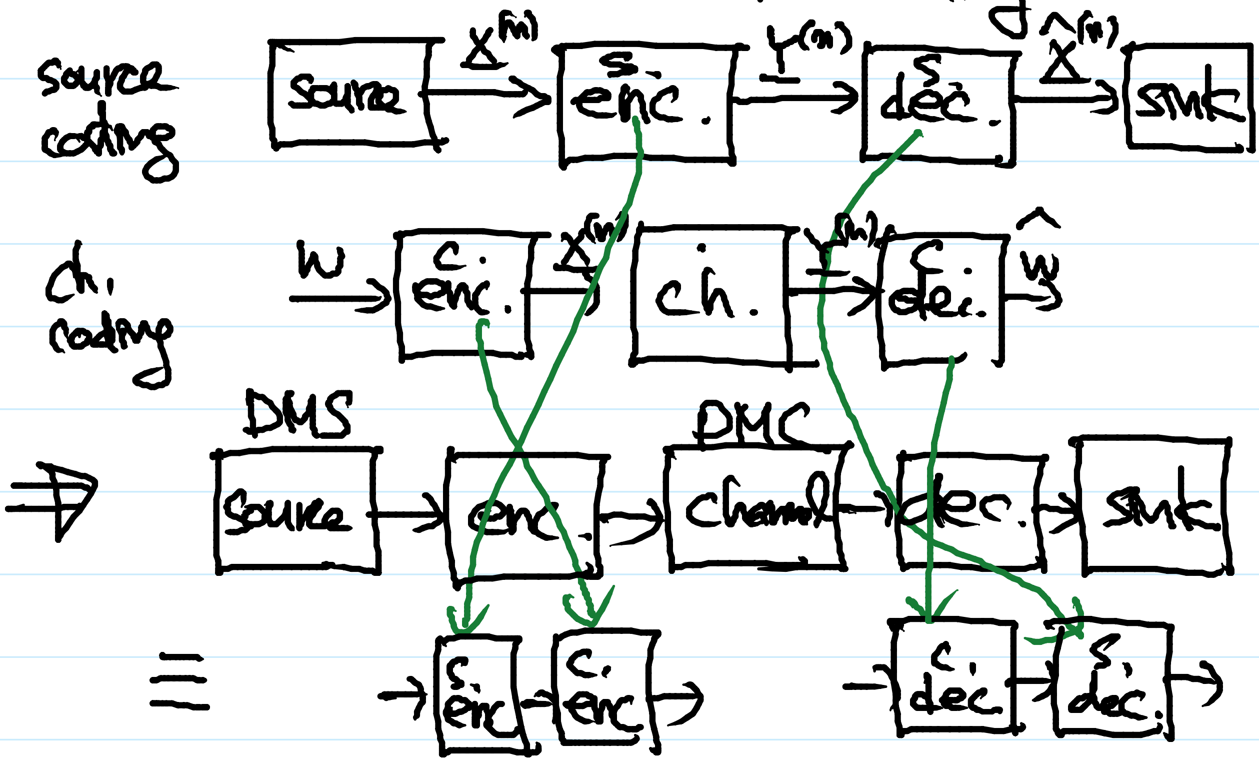


counter-intuitive.

- Today
  - Feedback capacity.
  - Joint source-channel coding



"layered design approach"

physical layer.

modular design

un-intuitive

source-channel separation theorem

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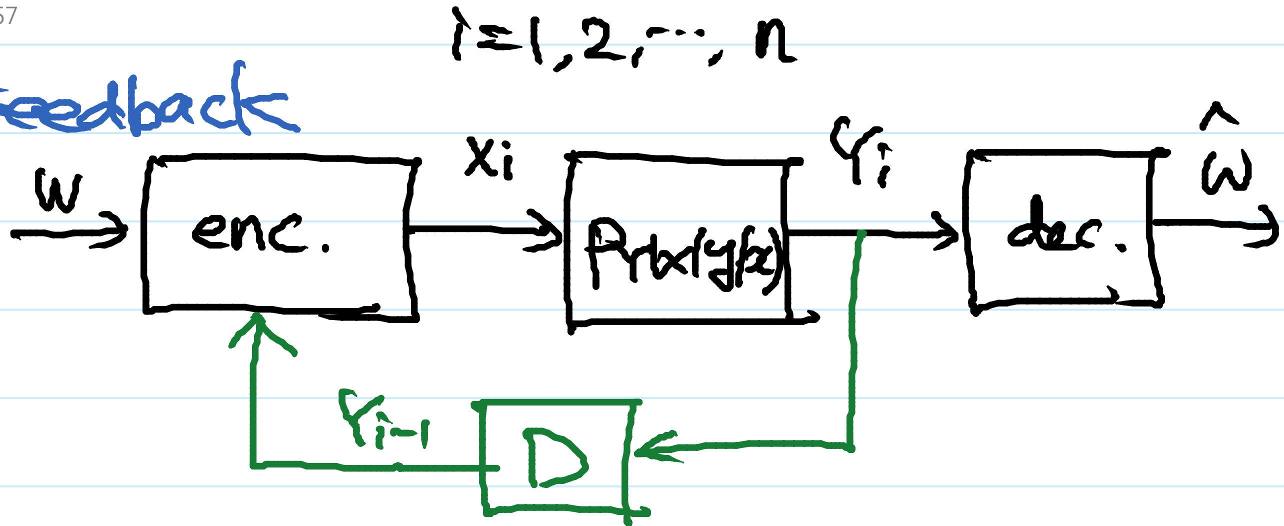
- Feedback



causal noiseless feedback of channel output.

An (M, n) feedback code

$X_1(W)$

$X_2(W, Y_1)$

$X_3(W, Y_1, Y_2)$

$\vdots$

$X_i(W, Y_1, Y_2, \cdots, Y_{i-1})$

$\vdots$

$X_n(W, Y_1, Y_2, \cdots, Y_{n-1})$

- Achievability

"$C = \max_{P_X(x)} I(X;Y)$ is achievable."

- Converse.

$$H(W) = H(W|\underline{Y}^{(n)}) + I(W; \underline{Y}^{(n)})$$
$$\leq 1 + nR \cdot P_{e,n} + H(\underline{Y}^{(n)}) - H(\underline{Y}^{(n)}|W)$$
$$= 1 + nR \cdot P_{e,n} + \sum_{i=1}^{n} H(Y_i) - \sum_{i=1}^{n} H(Y_i|W, Y_1, Y_2, \cdots, Y_{i-1})$$
$$\leq 1 + nR \cdot P_{e,n} + \sum_{i=1}^{n} H(Y_i) - \sum_{i=1}^{n} H(Y_i|\cancel{W, Y_1, Y_2, \cdots, Y_{i-1},} X_i)$$
$$= 1 + nR \cdot P_{e,n} + \sum_{i=1}^{n} I(X_i; Y_i)$$
$$\leq 1 + nR \cdot P_{e,n} + nC$$

$$\cancel{n}R \leq 1 + \cancel{n}R \cdot \cancel{P_{e,n}} + \cancel{n}C$$

$$\Rightarrow R \leq C$$

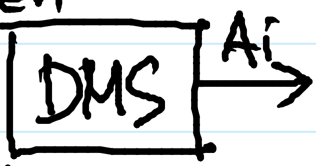
$$\Rightarrow C_{FB} \leq C.$$

Combined w/ the achievability,

$$C_{FB} = \max_{p_X(x)} I(X;Y).$$

• Joint Source-Channel Coding

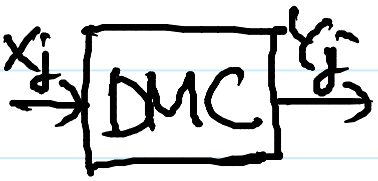
Given



H(A) [bits/source output]

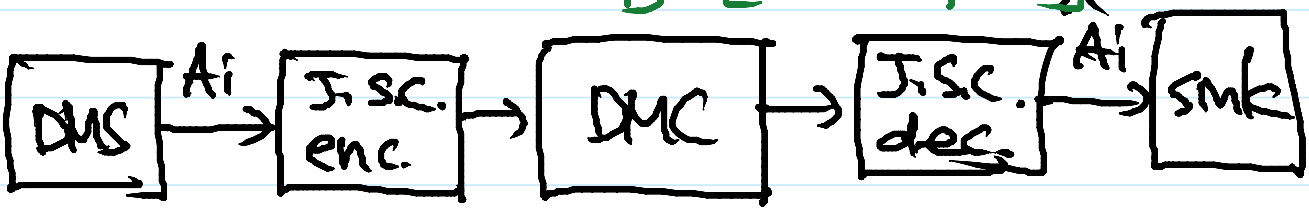
and

a [source outputs/sec]



$\max_{P_X(x)} I(X_j;Y_j)$ [bits/ch. use]

b [ch. use/sec]



$\rightsquigarrow$ H [bits/sec]

C [bits/sec]

• proof. $H \le C$

$H \ge C$

"If $H \le C$, then optimal joint SC code achieves $R \le H$ by separate S.C coding & $R > H$ is not achievable."

$P_{e,n} = P_e^{(n)}$

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$$\text{source} \xrightarrow{\underline{V}^{(n)}} \text{joint enc.} \xrightarrow{\underline{X}^{(n)}} \text{DMC} \xrightarrow{\underline{Y}^{(n)}} \text{joint dec.} \xrightarrow{\hat{\underline{V}}^{(n)}} \text{sink}$$

$$P_{e,n} = P_e^{(n)} \triangleq \Pr(\underline{V}^{(n)} \neq \hat{\underline{V}}^{(n)})$$

$$H(\underline{V}^{(n)}) = H(\underline{V}^{(n)} | \hat{\underline{V}}^{(n)}) + I(\underline{V}^{(n)} ; \hat{\underline{V}}^{(n)})$$

$$\le 1 + P_{e,n} \cdot \log|\mathcal{V}^n| + I(\underline{X}^{(n)} ; \underline{Y}^{(n)})$$

$$\le 1 + P_{e,n} \cdot n \cdot \log|\mathcal{V}| + nC$$

$$\cancel{n} H(V) \le 1 + \cancel{P_{e,n} \cdot n \log|\mathcal{V}|} + \cancel{n} C$$

$$H(V) \le C$$