Lec. 22

2013년 5월 6일 월요일
오후 8:50

Review

- zero-error variable-length
- abitrarily small-error fixed-length

} source coding

- Shannon's information measures

(ex)
$$H(X,Y) = H(X) + H(Y|X)$$

$$E\left\{\log \frac{1}{P_{XY}(X,Y)}\right\} = E\left\{\log \frac{1}{P_X(X)}\right\} + E\left\{\log \frac{1}{P_{Y|X}(Y|X)}\right\}$$

$$E\left\{\log \frac{1}{P_{Y|X}(Y|X)}\right\} = E\left[E\left\{\log \frac{1}{P_{Y|X}(Y|X)} \middle| X\right\}\right]$$

$$= E\{f(X)\}$$

where $f(x) = E\left\{\log \frac{1}{P_{Y|X}(Y|X)} \middle| X=x\right\}$

$$\triangleq H(Y|X=x)$$

$$D(P_X \| P_Y) \geq 0 \quad \ldots$$

$$H(X|\hat{X}) \leq \text{a function of } \Pr(X \neq \hat{X})$$

$$X \to Y \to Z \quad \Rightarrow \quad I(X;Y) \geq I(X;Z)$$

· Channel coding for DMC

$$C = \max_{p_X(x)} I(X;Y)$$

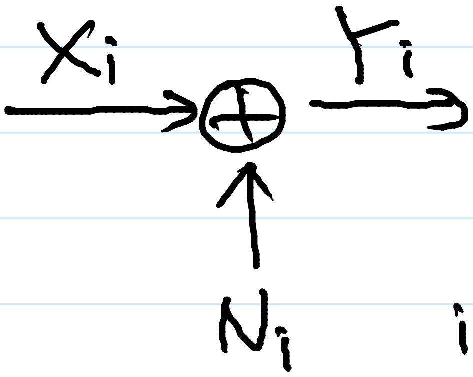
< $C_{FB} = C$ causal → simple. (extreme).

layered system noiseless

· Channel coding for CMC

AWGN channel

• Discrete-Time Additive White Gaussian Noise Channel



zero-mean i.i.d. Gaussian r.v.'s

$E\{N_i^2\} = \sigma^2 > 0$

$$C = \max_{f_X(x)} I(X;Y)$$

s.t. $E\{X^2\} \le P$

SNR.

$$C_{AWGN} = \frac{1}{2}\log\left(1+\frac{P}{\sigma^2}\right) \quad [\text{bits}/\text{c.u}]$$

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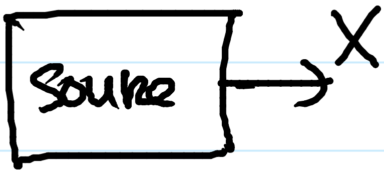
$$C = \sup_{f_{X^n}} I(X^n; Y^n)$$

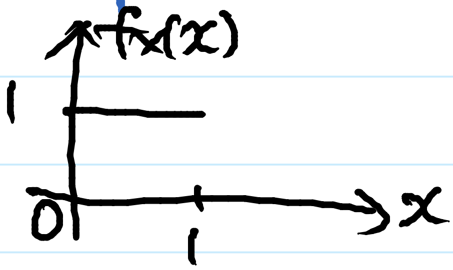
$$\text{s.t.} \quad E[X^2] \le P$$

- Given a r.v. X, X is ~~ft~~ of x
  - continuous if $F_X(x)$ is a continuous
  - discrete if $F_X(x)$ is piecewise const.
  - mixed if $F_X(x)$

where $F_X(x) \triangleq \Pr(X \le x)$

- 

simple



$1/3 = 0.333\ldots$

$1/\sqrt{2} = ?$

$H(X) \doteq +\infty$.

extension (consistency, backward compatibility)

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- Given a continuous r.v. X, we define a discrete r.v. X^Δ.

$$\Pr(a \le X \le b) = F_X(b) - F_X(a)$$

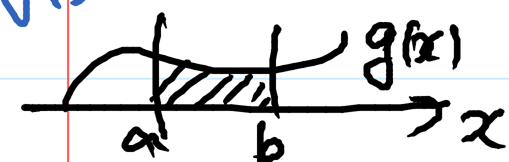
when X is continuous.

Now,

$$H(X^\Delta) = \sum_{i=-\infty}^{\infty} \left(F_X(\Delta_{i+1}) - F_X(\Delta_i)\right) \log \frac{1}{F_X(\Delta_{i+1}) - F_X(\Delta_i)}$$

$$\approx \sum_{i=-\infty}^{\infty} f_X(\Delta_i)\Delta \log \frac{1}{f_X(\Delta_i)\Delta}$$

Visualized



$$\int_a^b g(x)\,dx \triangleq \lim_{N\to\infty} \sum_{n=1}^{N} g\left(a + \frac{(n-1)}{N}\right)\frac{1}{N}$$

$$= \lim_{\Delta\to 0} \sum_{n=1}^{N} g(\Delta_n)\Delta$$

$$= \sum_{i=0}^{\infty} g(\Delta_i)\Delta$$

$$= \sum_{i=-\infty}^{\infty} f_X(\Delta i) \log \frac{1}{f_X(\Delta i)} \Delta +$$

$$\sum_{i=-\infty}^{\infty} f_X(\Delta i)\Delta \log \frac{1}{\Delta}$$

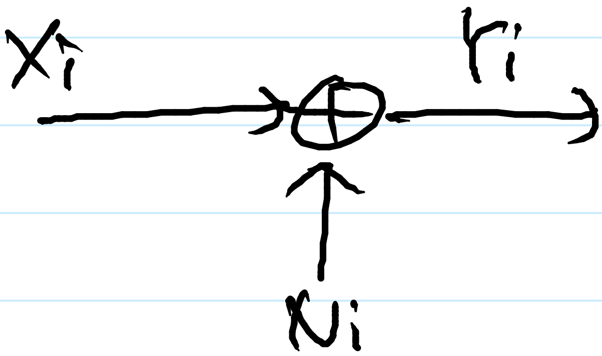
$$H(X^\Delta) \to \boxed{\int_{-\infty}^{\infty} f_X(x) \log \frac{1}{f_X(x)} dx} + \infty$$

as $\Delta \to 0$

$$\triangleq h(X)$$

$$\triangleq E\left\{\log \frac{1}{f_X(x)}\right\}$$

differential entropy.

- 

- Given jointly random two cont. r.v.'s X and Y, consider

$$I(X^\Delta; Y^\Delta) = H(X^\Delta) + H(Y^\Delta) - H(X^\Delta, Y^\Delta)$$

$$= h(X) + \sum_i f_X(\Delta_i)\Delta \log \frac{1}{\Delta}$$

$$+ h(Y) + \sum_j f_Y(\Delta_j)\Delta \log \frac{1}{\Delta}$$

$$- h(X,Y) - \sum_i \sum_j f_{X,Y}(\Delta_i, \Delta_j) \log \frac{1}{\Delta^2}$$

$$\left(F_{X,Y}(\Delta_{i+1}, \Delta_{j+1}) - F_{X,Y}(\Delta_i, \Delta_{j+1}) - F_{X,Y}(\Delta_{i+1}, \Delta_j) + F_{X,Y}(\Delta_i, \Delta_j) \right)$$

$$\simeq f_{X,Y}(\Delta_i, \Delta_j)$$

$$H(X,Y) \quad \sum_i \sum_j f_{X,Y}(\Delta_i, \Delta_j)$$

$$\log \frac{1}{\Delta^2} \quad \Delta^2 \qquad \simeq \int_\Delta f_{X,Y}(\Delta_i, \Delta_j)$$

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$$H(X^\Delta, Y^\Delta) \cong \sum_i \sum_j f_{X,Y}(\Delta i, \Delta j)\Delta^2 \log \frac{1}{\frac{f_{X,Y}(\Delta i, \Delta j)}{\Delta^2}}$$

$$= \sum_i \sum_j f_{X,Y}(\Delta i, \Delta j) \log \frac{1}{f_{X,Y}(\Delta i, \Delta j)} \Delta^2$$

$$+ \sum_i \sum_j f_{X,Y}(\Delta i, \Delta j) \log \frac{1}{\Delta^2} \cdot \Delta^2$$

$$= \iint f_{X,Y}(x,y) \log \frac{1}{f_{X,Y}(x,y)} \, dx\, dy$$

$$+ \sum_i \sum_j f_{X,Y}(\Delta i, \Delta j) \log \frac{1}{\Delta^2} \Delta^2$$

$$I(X^\Delta; Y^\Delta) \rightarrow h(X) + h(Y) - h(X,Y)$$

as $\Delta \rightarrow 0$ $\quad \triangleq I(X;Y)$

$$I(X;Y) = -E\left\{\log \frac{f_X(X) f_Y(Y)}{f_{X,Y}(X,Y)}\right\}$$

$$= h(X) - h(X|Y)$$

where $h(X|Y) = E\left\{\log \frac{1}{f_{X|Y}(X|Y)}\right\}$

$$= h(Y) - h(Y|X).$$

- $h(X)$, $h(Y)$, $h(X,Y)$
  $h(X|Y)$, $h(Y|X)$, $I(X;Y)$