Lec. 24

2013년 5월 9일 목요일

오후 1:52

- **Review**

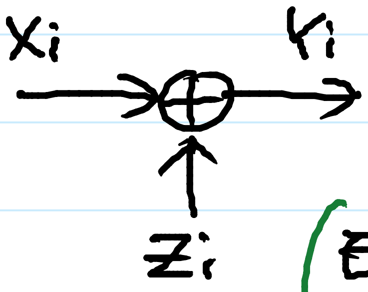
- DT (discrete-time) AWGN channel

$+$

independent of X_i

uncorrelatedness

$\Rightarrow$ independence



$$\begin{cases} E\{Z_i\} = 0, \ \forall i \\ E\{Z_i Z_j\} = \sigma_Z^2 \delta_{i,j} \end{cases}$$

- new problem — discover / given

search for a similar problem in your brain

similarities/commonalities, differences

simplify the problem keeping the essence

experience, lesson

increase the complexity

generalization

- Shannon's information measures

$$h(X) \triangleq E\left\{\log \frac{1}{f_X(X)}\right\}$$

- A.E.P. when $X_1, X_2, \cdots, X_n$ are r.i.d.

continuous r.v.'s

2013년 5월 9일 목요일

오후 1:53

- Informational channel capacity for cont. X & Y.

$$\tilde{C} \triangleq \max_{f_X(x)} I(X;Y)$$

W/o any constraint, $\tilde{C}$ could be $+\infty$.
Thus, we impose a β-admissibility constraint

$$E\{c(X)\} \leq \beta$$

for some constraint function $c(\cdot)$.

ex/ $c(x) = x^2$.

?

$$C = \sup_{f_X(x)} I(X;Y)$$

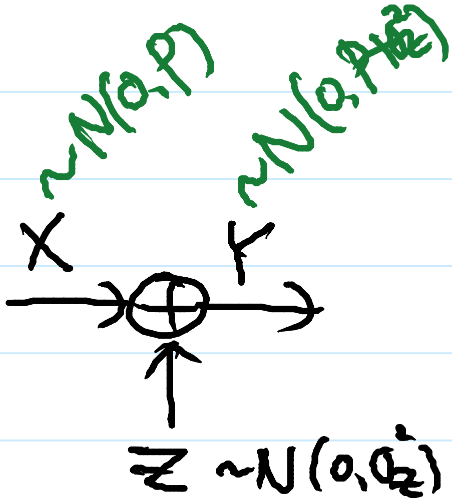
subject to $E\{c(X)\} \leq \beta$.

- $\Omega = \{f_X(x) : E\{c(X)\} \leq \beta\}$ may not be a convex set.
- $I(X;Y)$ may not be a concave ft of $f_X(x)$.

Page 02

2013년 5월 9일 목요일

오후 1:52



$\frac{1}{2}\log 2\pi e \sigma_z^2$

- $$C = \max_{f_X(x)} I(X;Y)$$

  s.t. $E\{X^2\} \le P$

represent

$$I(X;Y) = h(Y) - h(Y|X)$$
$$= h(Y) - h(X+Z|X)$$
$$= h(Y) - h(Z)$$
$$= h(Y) - \frac{1}{2}\log 2\pi e \sigma_z^2$$

where

$$E\{Y\} = E\{X+Z\} = E\{X\} \triangleq m.$$
$$Var\{Y\} = Var\{X+Z\} = Var\{X\} + \sigma_z^2$$

$$\le \frac{1}{2}\log\left(2\pi e\{Var\{X\} + \sigma_z^2\}\right) - \frac{1}{2}\log 2\pi e\sigma_z^2$$

where X is Gaussian

$$E\{X^2\} = m^2 + Var\{X\} \le P$$

$$\le \frac{1}{2}\log\{2\pi e\{(P-m^2) + \sigma_z^2\}\} - \frac{1}{2}\log 2\pi e\sigma_z^2$$

We choose $m=0$

$$\le \frac{1}{2}\log 2\pi e(P+\sigma_z^2) - \frac{1}{2}\log 2\pi e\sigma_z^2$$

$X \sim N(0, \bar{P})$. $\frac{1}{2} \log (1 + \frac{P}{\sigma_z^2})$ [bits/c.u.]

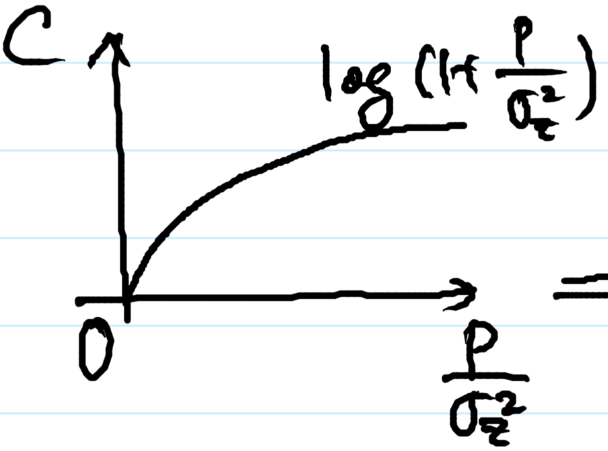
2013년 5월 9일 목요일
오후 1:52

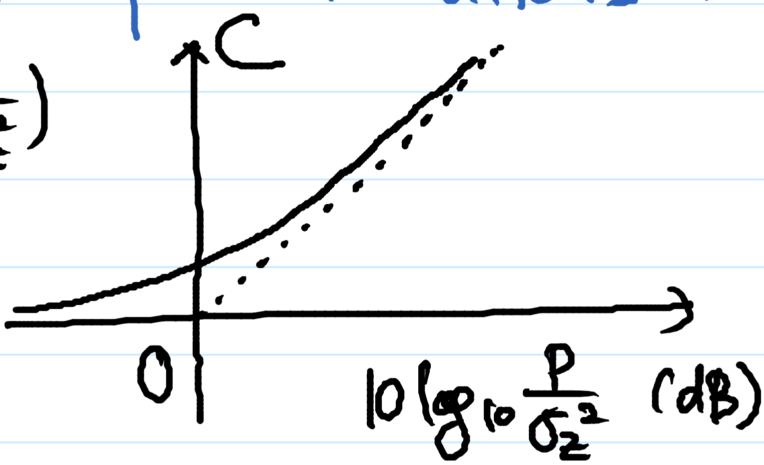
Informational Channel Capacity of power-constrained DT AWGN channel

average

$$C = \frac{1}{2}\log\left(1+\frac{P}{\sigma_z^2}\right) \text{ [bits/c.u.]}$$

Visualize! Which parameter affects how?





2013년 5월 9일 목요일
오후 1:52

- NCCT for average-power constrained DT AWGN channel.

Def. A rate R is achievable over a ~~memoryless~~ channel if there exists a sequence of codes $\{\mathcal{C}_k\}_k$ such that

channel

that satisfies the constraint

(i) $R_k \to R$

(ii) $\lambda_k \to 0$

Def. The operational channel capacity is defined as $\sup R$ where R is achievable.

Theorem. $\text{Operational} = \frac{1}{2}\log\left(1+\frac{P}{\sigma_z^2}\right)$

$E\{X^2\} \le P$

For example, given a codeword $\underline{x}_i^{(n)}$

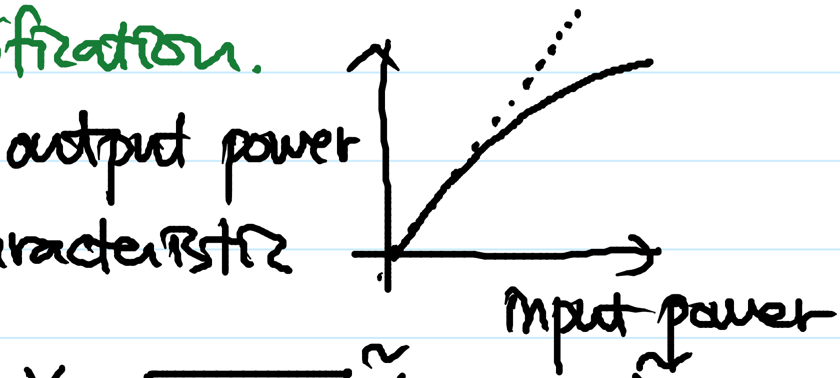
$x_i^{(n)} \triangleq (x_{i1}, \ldots, x_{in})$

$\underline{X}^{(n)} \triangleq (X_1, \quad \cdots\cdots \quad X_n)$

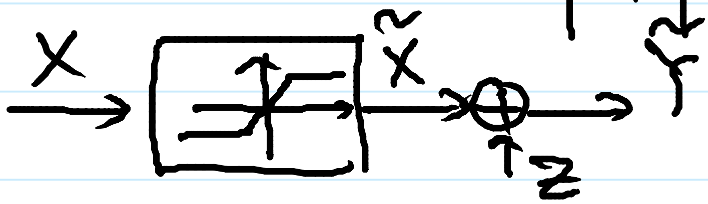
- Given $c(x)$,
  $E\{c(X)\} \le \beta$ is translated to
  $$\frac{1}{n}\sum_{i=1}^{n} c(x_{j,i}^{(n)}) \le \beta, \quad \forall j$$
  where $\underline{x}_j^{(n)}$ is the j-th codeword

However, this translation is an oversimplification.

PA characteristic



(cf) LNA



- $E\{X^2\} \le P \;\to\; \|\underbrace{x_i^{(n)}}\|^2 \le nP$

  i-th codeword of length n.

- Achievability

  Choose $f_X(x) = \frac{1}{\sqrt{2\pi(P-\epsilon')}} \exp\left(-\frac{x^2}{2(P-\epsilon')}\right)$

  Generate a random $(\lceil 2^{nR} \rceil, n)$ code $\mathcal{C}_n$

  Perform a modified typical set decoding

  Given $\{Y^{(n)} = y^{(n)}\}$,

  The decoder output $\hat{W}$ is given by

  $\hat{W} = m_i$ if

  $\boxed{\text{(i) } \|x_i^{(n)}\|^2 \le nP \text{ and}}$

  (ii) $(x_i^{(n)}, y^{(n)}) \in A_\epsilon^{(n)}$

  $(x_j^{(n)}, y^{(n)}) \notin A_\epsilon^{(n)}, \forall j \neq i$

  Otherwise declare error.

Page 07

2013년 5월 9일 목요일

오후 3:14

$$\mathcal{E}_n \rightarrow \Pr(\hat{W} \neq W \mid W = m_1, \mathcal{E}_n)$$

$$\vdots$$

$$\Pr(\hat{W} \neq W \mid W = m_{\lceil 2^{nR} \rceil}, \mathcal{E}_n)$$

$$\Pr(\hat{W} \neq W \mid W = m_i) = \Pr(\hat{W} \neq m_i \mid W = m_i)$$

$$\leq \Pr(\|\underline{X}^{(n)}\|^2 > nP)$$

$$+ \Pr((\underline{X}^{(n)}, \underline{Y}^{(n)}) \notin A_\varepsilon^{(n)})$$

$$+ (\lceil 2^{nR} \rceil - 1) \Pr((\tilde{\underline{X}}^{(n)}, \underline{Y}^{(n)}) \in A_\varepsilon^{(n)})$$

$$\leq \varepsilon + \varepsilon + 2^{-n(I(X;Y) - R - 3\varepsilon)}$$

$$R < I(X;Y) - 3\varepsilon$$

$\Rightarrow$ upper bound $\rightarrow 0$ as $n \rightarrow \infty$.