Lec. 26

2013년 5월 16일 목요일

오후 1:46

Review

- NCCT for DT AWGN channel

  Achievability: $R < \frac{1}{2}\log(1+\frac{P}{\sigma_Z^2})$ is achievable

  Converse: R is achievable then $R \le \frac{1}{2}\log(1+\frac{P}{\sigma_Z^2})$

$$\Rightarrow C_0 = \frac{1}{2}\log_2(1+\frac{P}{\sigma_Z^2}) \text{ [bits/ch.use]}$$

- Difference

  β-admissibility constraint

  $$E\{b(X)\} \le \beta, \quad E\{X^2\} \le P$$

  Achievability of R

  $$\frac{1}{n}\sum_{i=1}^{n} b(x_{i,j}^{(n)}) \le \beta, \quad \| \underline{x}_i^{(n)} \|^2 \le nP$$

$$C(\beta) = \sup_{f_X(x)} I(X;Y) \quad \text{s.t. } E[b(X)] \le \beta$$

Capacity-cost function.

continuous

$$H(W|\underline{Y}^{(n)}) \triangleq \int_{\underline{y}^{(n)}} H(W|\underline{Y}^{(n)}=\underline{y}^{(n)}) f_{\underline{Y}^{(n)}}(\underline{y}^{(n)}) d\underline{y}$$

$$I(W;\underline{Y}^{(n)}) \triangleq H(W) - H(W|\underline{Y}^{(n)})$$

$$\le I(\underline{X}^{(n)};\underline{Y}^{(n)})$$

116.pdf X_i's look like i.i.d. $p_X(x)$ ↑ optimal pmf.

- Gaussian Codebook

"Converse: If R is achievable, then $R \le C$."

(i) If we have a seq of codes that achieves $C-\varepsilon$. By the converse, this seq. of codes satisfies

$$h(\underline{Y}^{(n)}) - h(\underline{Y}^{(n)} | \underline{X}^{(n)})$$
$$\le \sum_{i=1}^{n} h(Y_i) - h(\underline{Y}^{(n)} | \underline{X}^{(n)})$$

where the equality holds iff Y_i's are independent.

$\Rightarrow$ If Y_i's & Z_i's are all indep. then X_i's are also indep.

(ii) $$\sum_{i=1}^{n} I(X_i ; Y_i) \le \sum_{i=1}^{n} \frac{1}{2} \log\left(1+ \frac{P_i}{\sigma_Z^2}\right)$$

$\Rightarrow$ If Y_i's & Z_i's are all indep. Gaussian, then X_i's are also indep. Gaussian

(iii) $$\sum_{i=1}^{n} \frac{1}{2} \log\left(1+ \frac{P_i}{\sigma_Z^2}\right) \le \frac{n}{2} \log\left(1+ \frac{P-\varepsilon}{\sigma_Z^2}\right)$$

(iii) $\sum_{i=1}^{n} \frac{1}{2} \log\left(1+\frac{P_i}{\sigma_Z^2}\right) \le \frac{n}{2} \log\left(1+\frac{P-\varepsilon}{\sigma_Z^2}\right)$

$\Leftrightarrow$ $P_i = P - \varepsilon \;\forall i$, i.e., X_i's are identical.

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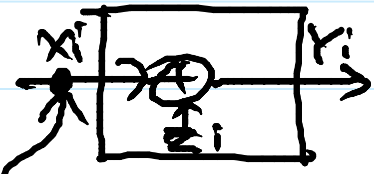
DT AWGN

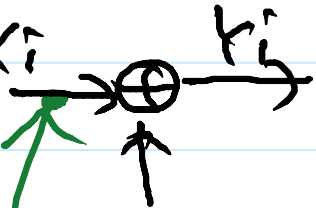


X_i's look like i.i.d. Gaussian r.variables

Thus, the near-optimal code is called the Gaussian codebook.

- Definition $\longrightarrow$ Existence, Uniqueness

ex/



P

$Z_i \sim N(0, \sigma_1^2)$

P

$\tilde{Z}_i \sim N(0, \sigma_2^2)$

$\sigma_1^2 < \sigma_2^2$

Compare!

$C_1 = \frac{1}{2}\log\left(1+\frac{P}{\sigma_1^2}\right)$

$C_2 = \frac{1}{2}\log\left(1+\frac{P}{\sigma_2^2}\right)$

X_i's $\sim$ i.i.d. $N(0, P)$

$\tilde{X}_i$'s $\sim$ i.i.d. $N(0, P)$

Rate of Gaussian Codebook #1 $>$ Rate of Gaussian Codebook #2

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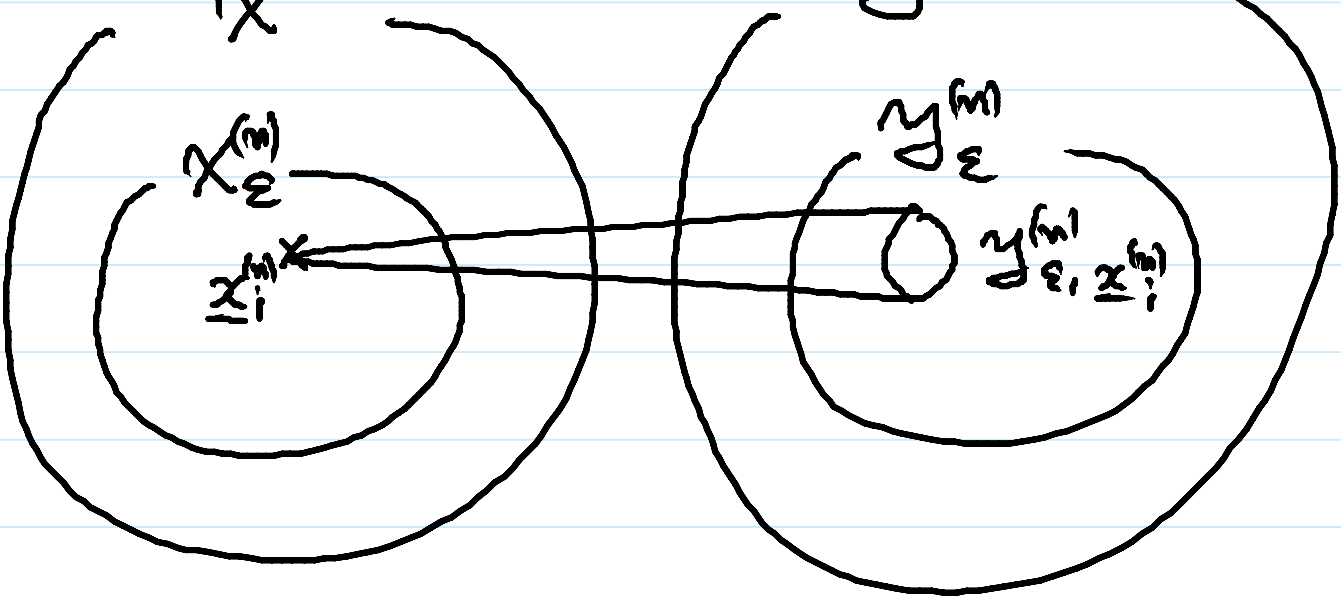
- Sphere Packing Argument

(i) DMC

(ii) DT AWGN

(i)



$|y_\varepsilon^{(n)}| \simeq 2^{nH(Y)}$

$|y_{\varepsilon,\underline{x}_i^{(n)}}^{(n)}| \simeq 2^{nH(Y|X)}$

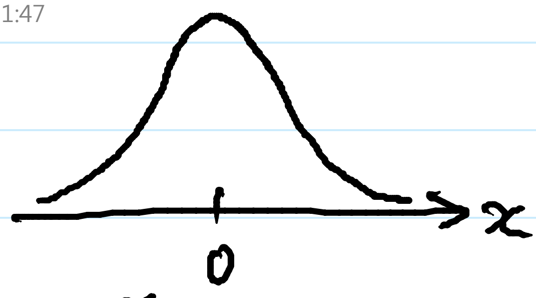
$Y = X + Z$, $X \sim N(0,P)$, $Z \sim N(0,\sigma_Z^2)$

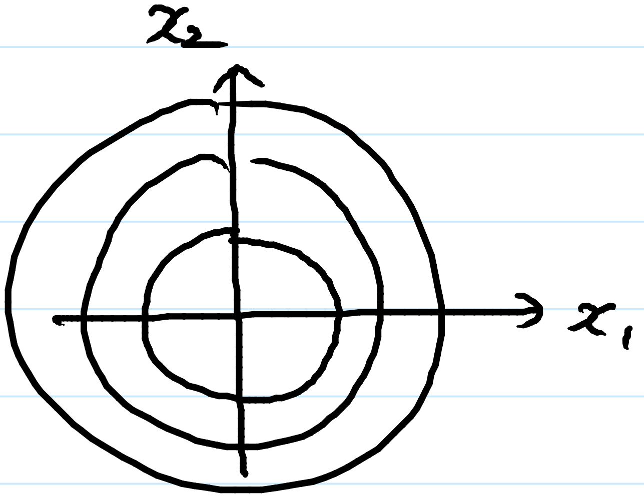
$$\Rightarrow \frac{\log_2 2^{nI(X;Y)}}{n} = I(X;Y)$$

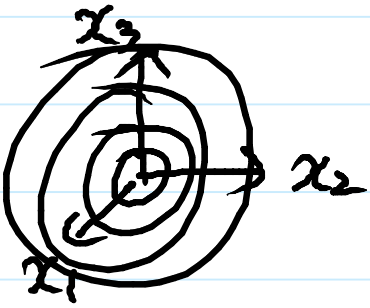
(ii) $\mathrm{Vol}(y_\varepsilon^{(n)}) \simeq 2^{nh(Y)}$

$\mathrm{Vol}(y_{\varepsilon,\underline{x}_i^{(n)}}^{(n)}) \simeq 2^{nh(Y|X)}$

$$\Rightarrow \frac{\log_2 2^{nI(X;Y)}}{n} = I(X;Y) \leq \frac{1}{2}\left(1+\frac{P}{\sigma_Z^2}\right)$$

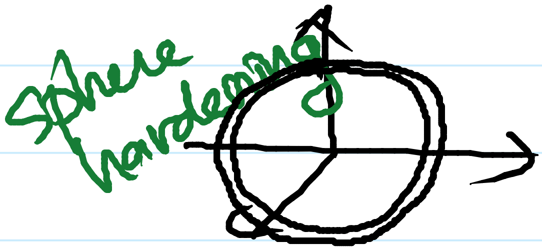






...

$$\|\underline{X}^{(n)}\|^2 \to nP$$



$\|\underline{x}\| = 1$

$x_1^2 + x_2^2 = 1.$

(ii) $\underline{Y}^{(n)} = \underline{x}_i + \underline{Z}^{(n)}$ $\quad \|\underline{Y}^{(n)} - \underline{x}_i\|_2^2 \simeq n\sigma_Z^2 \quad \sqrt{n\sigma_Z^2}$

$x_1^2 + x_2^2 + \cdots + x_n^2 = n$

(iii) $\underline{Y}^{(n)} = \underline{X}^{(n)} + \underline{Z}^{(n)}$ $\quad \|\underline{Y}^{(n)}\|^2 \simeq n(P+\sigma_Z^2)$

$\sqrt{n(P+\sigma_Z^2)}$

(iv) $\dfrac{Vol}{Vol} \simeq \dfrac{\sqrt{n(P+\sigma_Z^2)}^{\,n}}{\sqrt{n\sigma_Z^2}^{\,n}}$

$$\frac{\log \dfrac{\sqrt{n(P+\sigma_Z^2)}^{\,\cancel{n}}}{\sqrt{n\sigma_Z^2}^{\,\cancel{n}}}}{\cancel{n}} = \frac{1}{2}\log\left(1+\frac{P}{\sigma_Z^2}\right)$$

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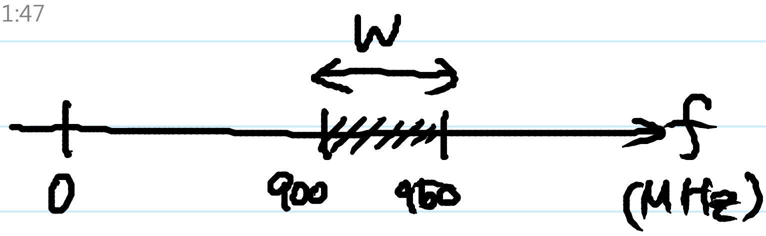
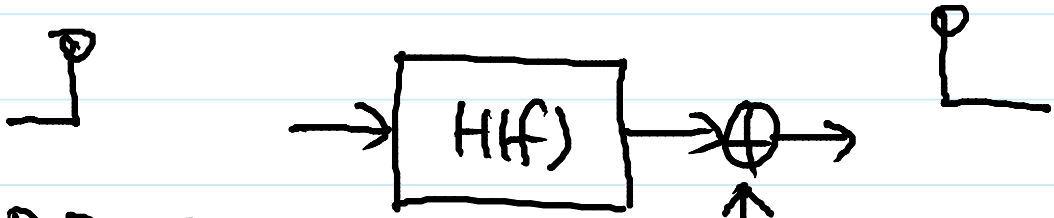




P [Watt]

CT freq. selective channel

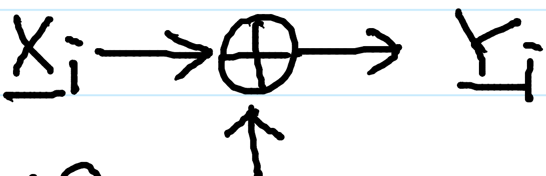
N(t) AWGN

2-sided PSD is $\frac{N_0}{2}$ [Watt/Hz].

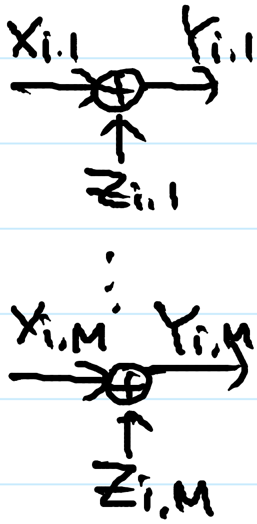
If flat, then

$$C = W \log\left(1 + \frac{P}{N_0 W}\right) \quad [\text{bits/sec}]$$

- Simplification
  - DT flat — vector
  - DT selective
  - CT flat



$\equiv$



$E[\|\underline{X}_i\|^2] \le P$.

$\underline{Z}_i \sim N(\underline{0}, \cancel{R})$ i.i.d. $\cancel{\sigma^2 I}$

$\text{diag}(\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2)$

< Parallel Gaussian channel
Gaussian product channel

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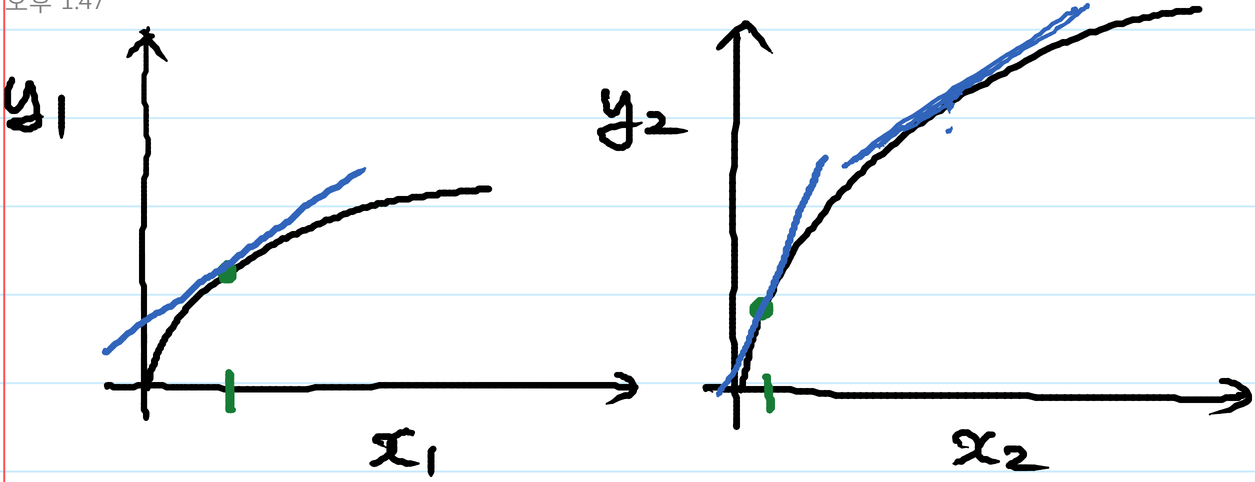

→ slopes should be equal!

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오후 3:11

- $$C_I = \max_{f_{\underline{X}}(\underline{x})} I(\underline{X};\underline{Y})$$

  s.t. $E\{\|\underline{X}\|^2\} \le P$

  where $\underline{Y} = \underline{X} + \underline{Z}$ &

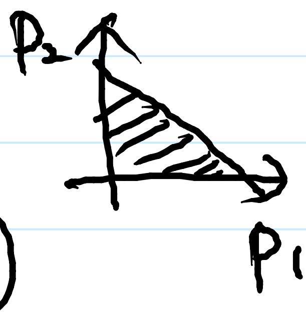
  $\underline{Z} \sim N(\underline{0}, \mathrm{diag}(\sigma_1^2, \sigma_2^2, \cdots, \sigma_k^2))\}$

- $$\begin{aligned} I(\underline{X};\underline{Y}) &= h(\underline{Y}) - h(\underline{Y}|\underline{X}) \\ &\le \sum_{i=1}^{k} h(Y_i) - h(\underline{Z}) \\ &= \sum_{i=1}^{k} h(Y_i) - h(Y_i|X_i) \\ &= \sum_{i=1}^{k} I(X_i;Y_i) \\ &\le \sum_{i=1}^{k} \frac{1}{2}\log\left(1+\frac{P_i}{\sigma_i^2}\right) \end{aligned}$$

  Y_i's are independent.

  $\Rightarrow$ X_i's are independent.

  X_i's are Gaussian w/ variance P_i



- $$C_I = \max_{(P_i)_i} \sum_{i=1}^{k} \frac{1}{2}\log\left(1+\frac{P_i}{\sigma_i^2}\right)$$

  s.t. $0 \le P_i, \forall i, \sum_{i=1}^{k} P_i \le P$