Lec. 28

오후 2:00

Random process ←

→ Markov process

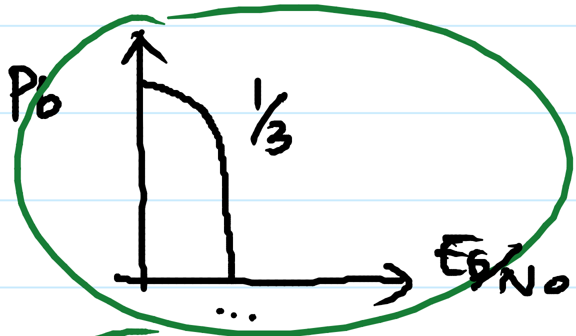
Peebles vs. Leon-Garcia

· Review

i.i.d. source

- source coding theorem ←
- channel coding theorem ←
- rate distortion theorem ⇐

- data compression
  - lossless
    - ZE
    - a. small error
  - lossy
- data transmission
  - DMC
  - CMC : DT-AWGN



Detection & $D(p_x \| p_r)$

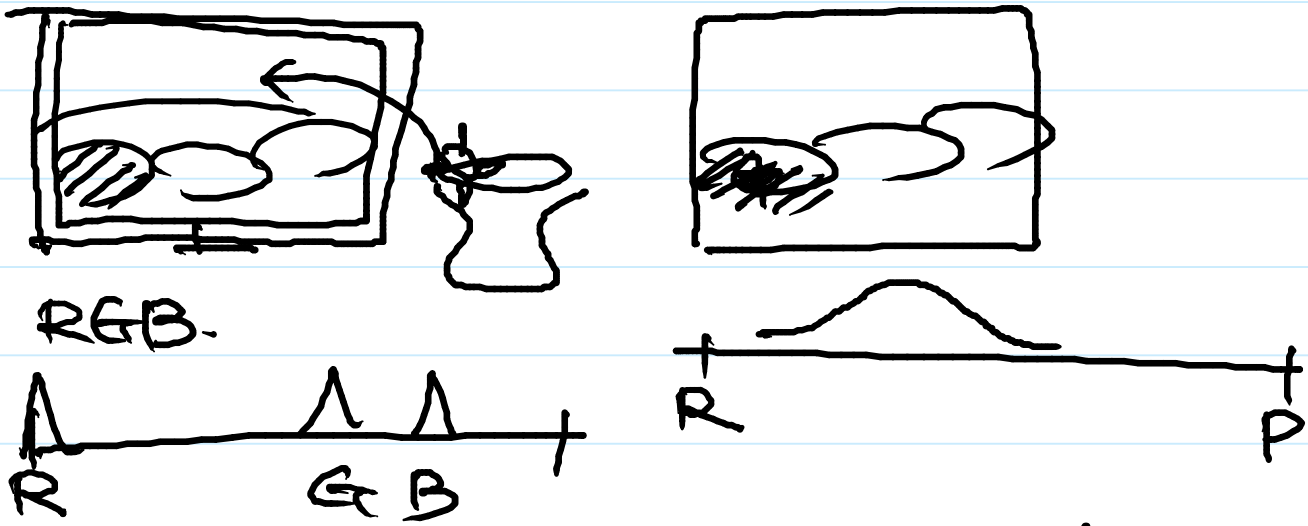
- Problem

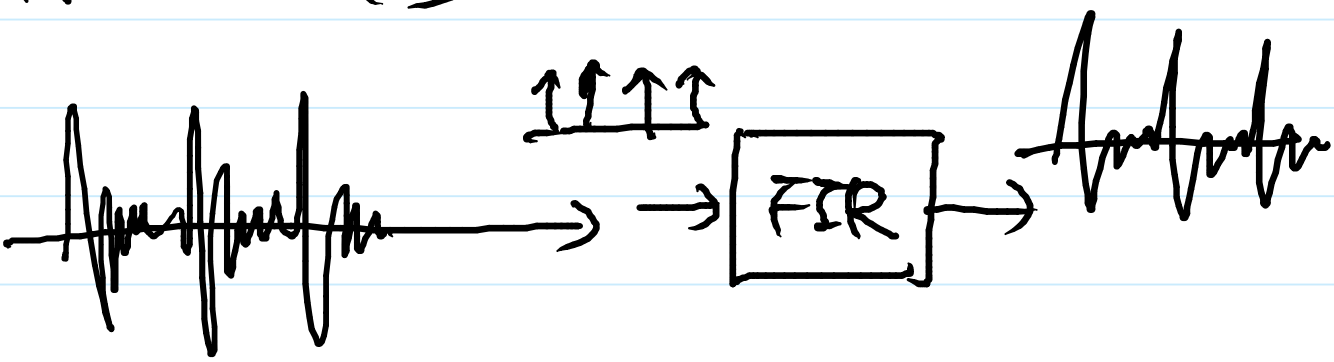
minimization of
# of bits/source output

R

$$\min_{C} E\{l(X)\}$$

s.t. $\Pr(X^{(n)} \neq \hat{X}^{(n)}) = 0$

- 

- 

- mp3. psychoacoustic signal processing

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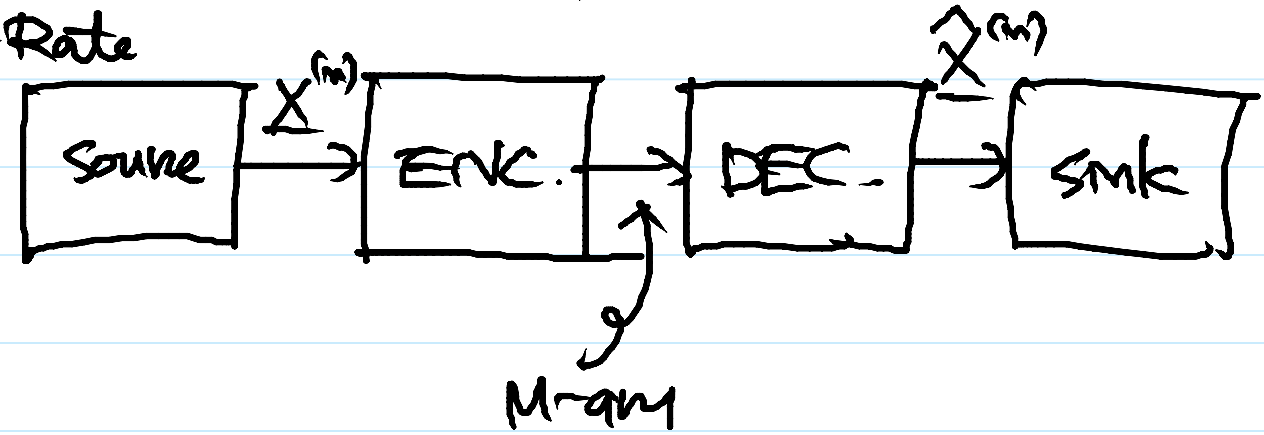
- Lossy compression

$\min R$
s.t. $\text{loss} \le L$

— Rate



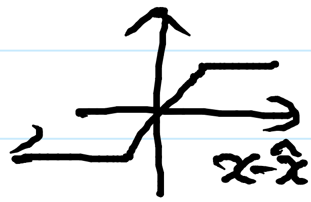
Rate R is defined as

$$R \triangleq \frac{\log_2 M}{n} \text{ [bits/source output]}$$

$\mathcal{X}$ $\hat{\mathcal{X}}$

— Loss

$d(x, \hat{x}) = (x-\hat{x})^2, \ |x-\hat{x}|,$



↑
distance metric

⋮

$\mathcal{X}$: source alphabet
$\hat{\mathcal{X}}$: reconstruction alphabet

$$d(x^{(n)}, \hat{x}^{(n)}) = \frac{1}{n}\sum_{i=1}^{n} d(x_i, \hat{x}_i)$$

- We are interested in

$n' \ll n$

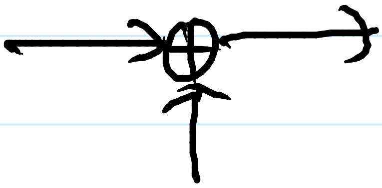
$$d(x,\hat{x}) = d_H(x,\hat{x}) = \begin{cases} 0, & \text{if } x=\hat{x} \\ 1, & \text{if } x \neq \hat{x} \end{cases}$$

distortion measure

Hamming distance (metric)

$$d(\underline{X}^{(n)}, \hat{\underline{X}}^{(n)}) = \frac{1}{n}\sum_{i=1}^{n} d_H(X_i,\hat{X}_i)$$
$$= \overline{P_b}$$

- Sample Problem



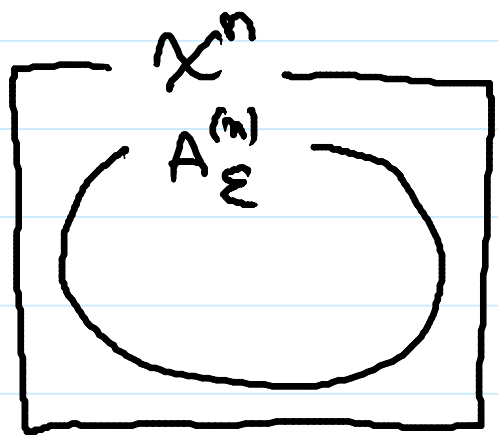
DT AWGN

Q. $\overline{P_b} > 0$, $R \uparrow$.

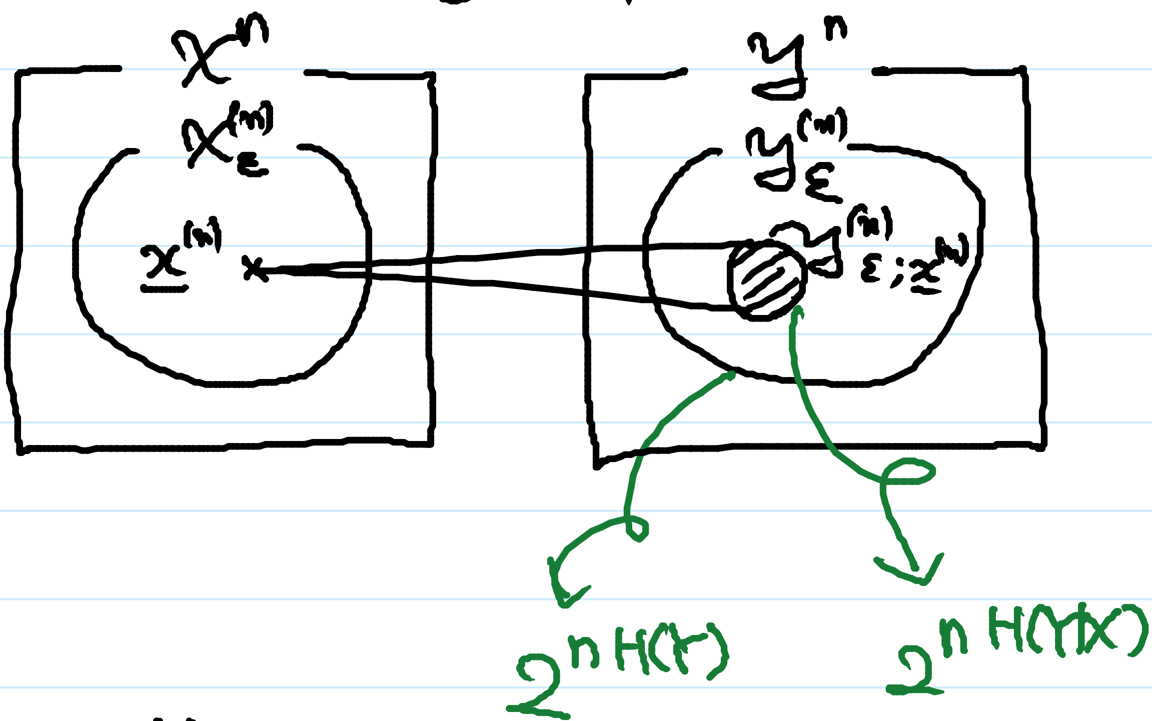
cf) $\overline{P_b} \simeq 0$, $R < \frac{1}{2}\log\left(1+\frac{P}{\sigma_z^2}\right)$

- Sketch of the proof
  - Source Coding

AEP.



  - Channel Coding $p_{Y|X}(y|x)$



$$\frac{2^{nH(Y)}}{2^{nH(Y|X)}} = 2^{nI(X;Y)}$$

$\Downarrow$ $$C_I = \max_{p_X(x)} I(X;Y)$$

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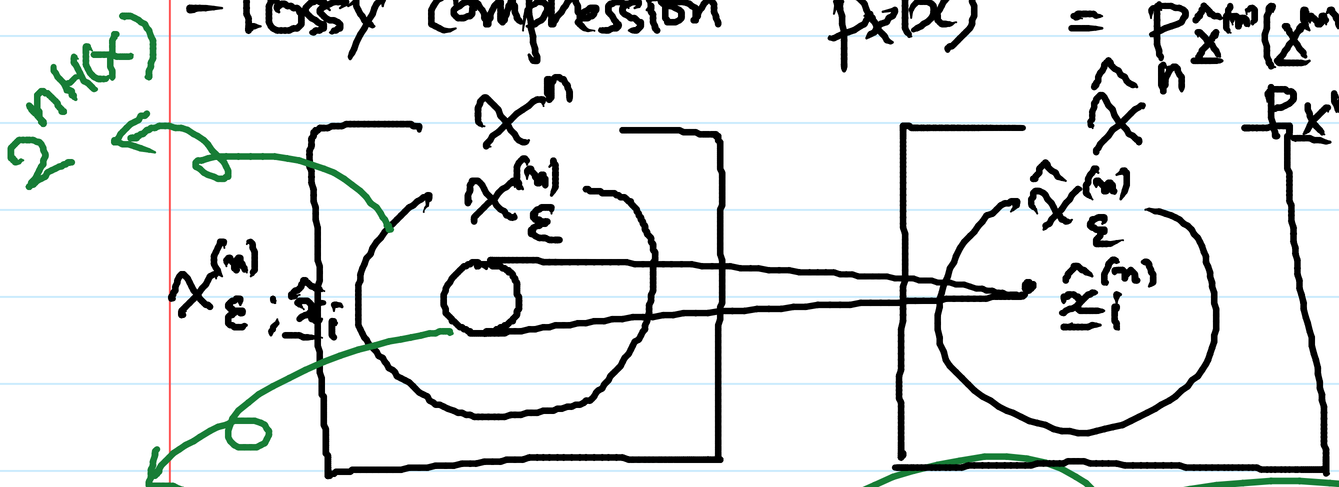
- lossy compression

Given $P_X(x)$, choose $P_{\hat{X}|X}(\hat{x}|x)$

$$P_{X^{(n)},\hat{X}^{(n)}}(x^{(n)},\hat{x}^{(n)}) = P_{\hat{X}^{(n)}|X^{(n)}}(\hat{x}^{(n)}|x^{(n)})\,P_{X^{(n)}}(x^{(n)})$$

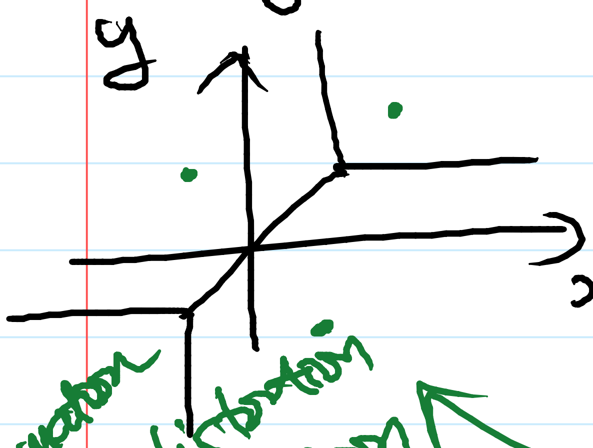


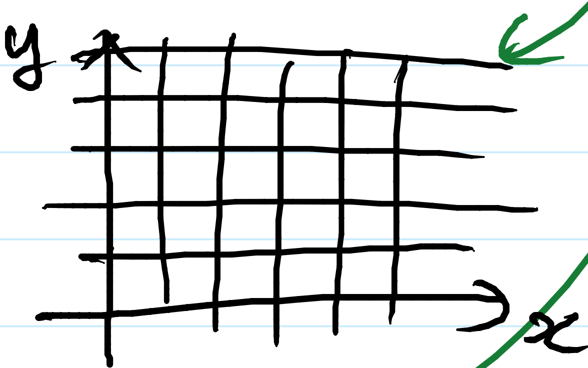
$\mathbb{R} \to \mathbb{Q}$. scalar quan.

$\mathbb{R}^2 \to \hat{\mathbb{R}}^2$. Vector "

< Discrete-time SP \ Quantization

Digital SP





$2^{nI(X;\hat{X})}$

information rate-distortion definition

$$\therefore R_I(D) = \min_{P_{\hat{X}|X}(\hat{x}|x)} I(X;\hat{X})$$

s.t. $E\{d(X,\hat{X})\} \le D$

Sphere Covering Argument.

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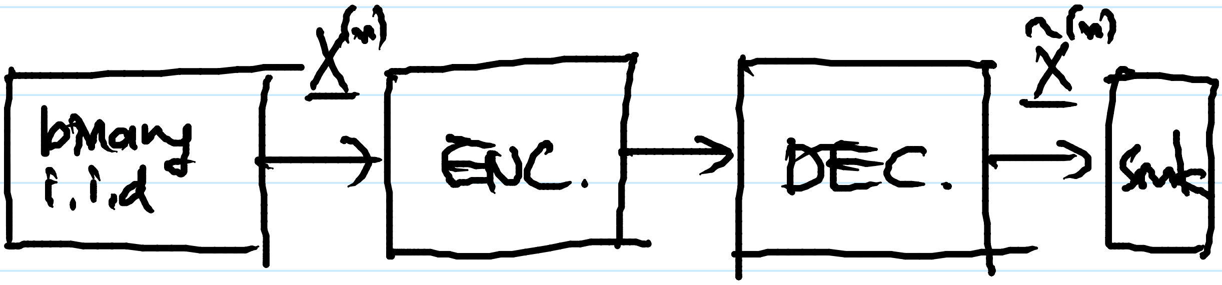
- Example.



$Pr(X_i=0)$
$=Pr(X_i=1)$
$=1/2$

$d(x,\hat{x}) \triangleq d_H(x,\hat{x})$

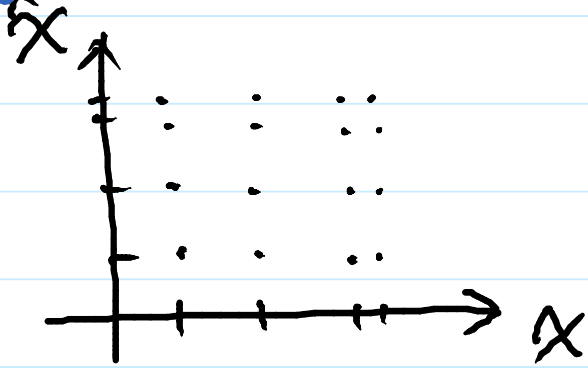
$$\min_{P_{\hat{X}|X}} I(X;\hat{X})$$
$$\text{s.t. } E\{d(X,\hat{X})\} \le P_b$$

$\Rightarrow$ Too good to be true if

(i) $\{P_{\hat{X}|X} : E\{d(X,\hat{X})\} \le D\}$ cvx

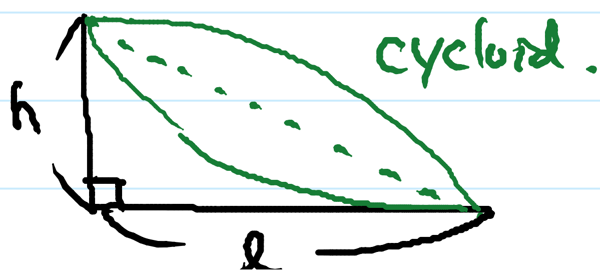
(ii) $I(X;\hat{X})$ is a cvx function of $P_{\hat{X}|X}(\hat{x}|x)$.

[commonality difference.



Calculus of Variation

변분법.



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- Def. A rate R is achievable if there exists a sequence of (M_k, n_k) rate distortion codes $\mathcal{C}_{n_k}$. such that

(i) $R_k \triangleq \frac{\log_2 M_k}{n_k} \to R$ as $k \to \infty$

(ii) $\lim_{k\to\infty} E\{d(\underline{X}^{(n_k)}, \hat{\underline{X}}^{(n_k)})\} \leq D$

$S = \{x_1, x_2, \cdots, x_k, \cdots\}$

$\sup S$

$\limsup_n x_n \cdots$