Lec. 30

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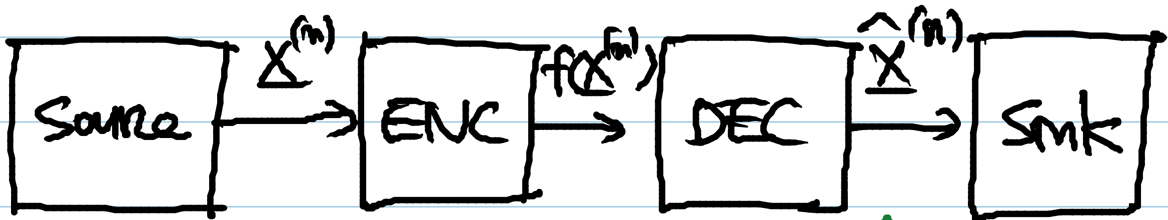
- Review
  - data compression — 500 bits → 300 bits, → 200 bits — ↓ [bits/source symbol]
  - data transmission — ↑ [bits/ch. use]



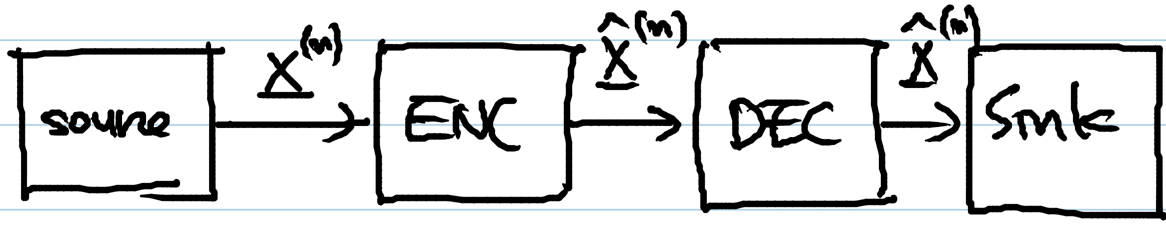
$\mathcal{X}$ source alphabet

$1 \sim M$

$\hat{\mathcal{X}}$ reconstruction alphabet

$$d(x,\hat{x}) : \mathcal{X}\times\hat{\mathcal{X}} \to \mathbb{R}_+$$

distortion measure



- Rate distortion theorem
  - Achievability: If $R > R_I(D)$ then $\exists$ a sequence of $(\lceil 2^{n_k R}\rceil, n_k)$ rate-distortion codes $\mathcal{C}_k$ s.t.
$$\lim_{k\to\infty} E\{d(X^{(n_k)},\hat{X}^{(n_k)})\} \le D$$

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~~R is achievable w/~~

$\cancel{\lim_{(n)\to\infty} E\{d(X^{(n)}, \hat{X}^{(n)})\} \leq D.}$

$\Longrightarrow$ (R,D) is achievable.

Converse If (R,D) is achievable then

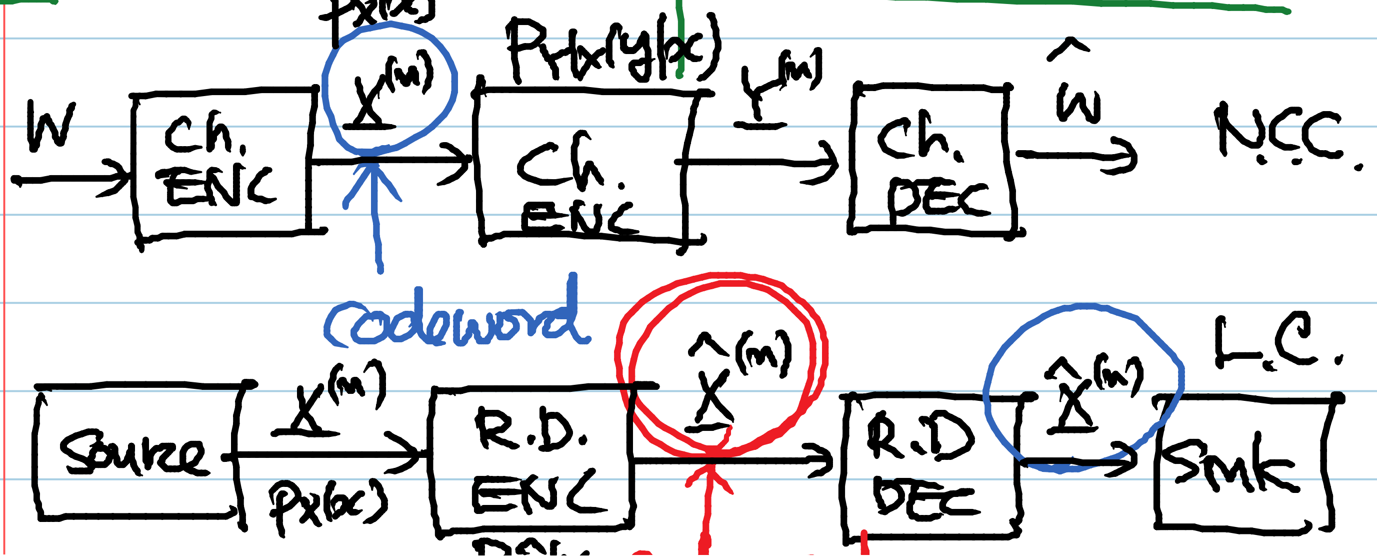
$$R > R_I(D)$$

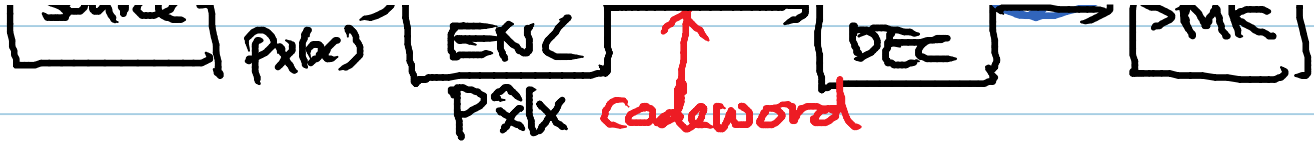
commonality

difference

- Achievability

Commonality	Difference
random coding - randomly generate codewords Typical set	- Use $p_{\hat{X}}(\hat{x})$. Typical set encoding




Px(x)
ENC
DEC
codeword

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- Typical set encoding rule

codewords
$\underline{\hat{x}}_1^{(n)}$
$\underline{\hat{x}}_2^{(n)}$
$\vdots$
$\underline{\hat{x}}_M^{(n)}$

$$(\underline{x}^{(n)}, \underline{\hat{x}}_i^{(n)}) \in A_{d,\varepsilon}^{(n)}$$

Use smallest index.

$p_{\hat{x}}(\hat{x})$

- Distortion ε-typical set

Given $p_X(x)$ and choice of $p_{\hat{X}|X}(\hat{x}|x)$,

$$A_{d,\varepsilon}^{(n)} = \{ (\underline{x}^{(n)}, \underline{\hat{x}}^{(n)}) \in \mathcal{X}^n \times \hat{\mathcal{X}}^n :$$

(i) $\left| -\frac{1}{n}\sum_i \log p_X(\underline{x}_i^{(n)}) - H(X) \right| \le \varepsilon,$

(ii) $\left| -\frac{1}{n}\sum_i \log p_{\hat{X}}(\underline{\hat{x}}_i^{(n)}) - H(\hat{X}) \right| \le \varepsilon,$

(iii) $\left| -\frac{1}{n}\sum_i \log p_{X,\hat{X}}(\underline{x}_i^{(n)}, \underline{\hat{x}}_i^{(n)}) - H(X,\hat{X}) \right| \le \varepsilon,$

and

(iv) $\left| \frac{1}{n}\sum_i d(\underline{x}_i^{(n)}, \underline{\hat{x}}_i^{(n)}) - E\{d(X,\hat{X})\} \right| \le \varepsilon \}$

- If $(\underline{x}^{(n)}, \hat{\underline{x}}^{(n)}) \in A_{d,\varepsilon}^{(n)}$, then $(\underline{x}^{(n)}, \hat{\underline{x}}^{(n)})$

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is called distortion typical.

- Lemma.

  If $(X^{(n)}, \hat{X}^{(n)})$ is generated by using $p_{X,\hat{X}}(x,\hat{x})$ independently n times,

  then $\Pr\{(X^{(n)}, \hat{X}^{(n)}) \in A^{(n)}_{d,\varepsilon}\} \to 1$ as $n \to \infty$, $\forall \varepsilon$.

(proof)

$\Pr\{(\quad) \notin A^{(n)}_{d,\varepsilon}\}$

$\leq \Pr($ $X^{(n)}$ is not typical$)$

$+\Pr($ $\hat{X}^{(n)}$ " " " $)$

$+\Pr($ $(X^{(n)}, \hat{X}^{(n)})$ " " " $)$

$+\Pr\left(\left| d(X^{(n)}, \hat{X}^{(n)}) - E\{d(X,\hat{X})\} \right| > \varepsilon \right)$

$\leq \varepsilon' + \varepsilon' + \varepsilon' + \varepsilon'$

- Lemma

  Given $p_{X,\hat{X}}(x,\hat{x})$, for all $(x^{(n)}, \hat{x}^{(n)}) \in A^{(n)}_{d,\varepsilon}$ we have

$\simeq 1$

$$p(\underline{\hat{x}}^{(n)}) \geq p(\underline{\hat{x}}^{(n)} \mid \underline{x}^{(n)})\, 2^{-n(I(X;\hat{X})+3\varepsilon)}$$

$$Pr(\underline{\hat{X}}^{(n)} = \underline{\hat{x}}^{(n)}) \qquad Pr(\underline{\hat{X}}^{(n)} = \underline{\hat{x}}^{(n)} \mid \underline{X}^{(n)} = \underline{x}^{(n)})$$

$$\Leftrightarrow p(\underline{x}^{(n)})\, p(\underline{\hat{x}}^{(n)}) \geq p(\underline{x}^{(n)}, \underline{\hat{x}}^{(n)})\, 2^{-n(I(X;\hat{X})+3\varepsilon)}$$

$$Pr(\underline{X}^{(n)} = \underline{x}^{(n)}) \geq 2^{-n(H(X)+\varepsilon)}$$

$$Pr(\underline{\hat{X}}^{(n)} = \underline{\hat{x}}^{(n)}) \geq 2^{-n(H(\hat{X})+\varepsilon)}$$

$$Pr\left((\underline{X}^{(n)}, \underline{\hat{X}}^{(n)}) = (\underline{x}^{(n)}, \underline{\hat{x}}^{(n)})\right) \leq 2^{-n(H(X,\hat{X})-\varepsilon)}$$