Lec. 32

2013년 5월 30일 목요일

오후 1:52

- Review
  - data compression
    - lossless
      - variable-length
      - fixed-length
    - lossy
  - data transmission

- $$\text{DMS} \xrightarrow{\underline{U}^{(k)}} \text{ENC} \xrightarrow{\underline{X}^{(n)}} \text{DMC} \xrightarrow{\underline{Y}^{(n)}} \text{DEC} \xrightarrow{\hat{\underline{U}}^{(k)}} \text{Sink}$$

We want $E\{d(\underline{U}^{(k)}, \hat{\underline{U}}^{(k)})\} \le D$

Joint Source-Channel Coding w/ Distortion

- cf) w/o distortion

$$\text{DMS} \xrightarrow{H(U)} \qquad\qquad \xrightarrow{X} \text{DMC} \xrightarrow{Y} \quad C$$

[bits/source output] [bits/channel use]

$$kH(U) \le nC$$

Error-free transmission is possible if

$\exists\ (k_i, n_i)$ joint source-channel code s.t. $\lim_{i \to \infty} \frac{k_i}{n_i} \leq \frac{C}{H(U)}$, $n_i \to \infty$

[source outputs / channel use]

- Def.

  Given a pair of a DMS & a DMC, $\boxed{\&D}$,
  a rate $\boxed{r \text{ is achievable}}$ by a sequence of (k_i, n_i) joint source-channel codes w/ distortion if

  (i) $\lim_{i\to\infty} \frac{k_i}{n_i} = r$ where $n_i, k_i \to \infty$ as $i \to \infty$.

  (ii) $\lim_{i\to\infty} E\{d(\underline{U}^{(k_i)}, \hat{\underline{U}}^{(k_i)})\} \le D$

- Claim

  A rate r is achievable iff

  $$r R(D) \le C$$

  Note that $R(D) \le H(U)$ in general.

- Achievability

  $$\boxed{\text{DMS}} \xrightarrow{k_i} \boxed{\text{R.D ENC}} \to \boxed{\text{Ch. ENC}} \xrightarrow{n_i} \boxed{\text{DMC}} \to \boxed{\text{Ch. DEC}} \to \boxed{\text{R.D DEC}} \to \boxed{\text{Sink}}$$

  $k_i R(D)$ [bits] $\le$ $n_i C$ [bits]

$$\Rightarrow \quad \frac{k_i}{n_i} R(D) \leq C \quad \Rightarrow \quad r R(D) \leq C$$

- Converse

From the DPI, we have

$$k_i R(D) \le I(\underline{U}^{(k_i)}, \hat{\underline{U}}^{(k_i)}) \le I(\underline{X}^{(n_i)}, \underline{Y}^{(n_i)}) \le n_i C$$

(LHS) $I(\underline{U}^{(k_i)}, \hat{\underline{U}}^{(k_i)}) = H(\underline{U}^{(k_i)}) - H(\underline{U}^{(k_i)} | \hat{\underline{U}}^{(k_i)})$

$$\geq \sum_k H(U_k) - \sum_k H(U_k | \hat{U}_k)$$

$$= \sum_k I(U_k ; \hat{U}_k)$$

$$\vdots$$

$$\geq k_i R(D)$$

(RHS) $I(\underline{X}^{(n_i)}; \underline{Y}^{(n_i)}) = H(\underline{Y}^{(n_i)}) - H(\underline{Y}^{(n_i)} | \underline{X}^{(n_i)})$

$$\leq \sum_n H(Y_n) - \sum H(Y_n | X_n)$$

$$= \sum_n I(\underline{X}^{(n_i)} ; \underline{Y}^{(n_i)})$$

$$\leq n_i C$$

$$\therefore \quad k_i R(D) \leq n_i C$$

$$\Rightarrow \quad r\, R(D) \leq C.$$

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strictly band-limited

- Spectral Efficiency Plane for CT AWGN channel

E_b	bit energy	[Joule], [Joules/bit]
N_o	one-sided PSD	[watts/Hz]
W	bandwidth	[Hz]
R	transmission rate	[bps]

$\frac{R}{W}$ spectral efficiency

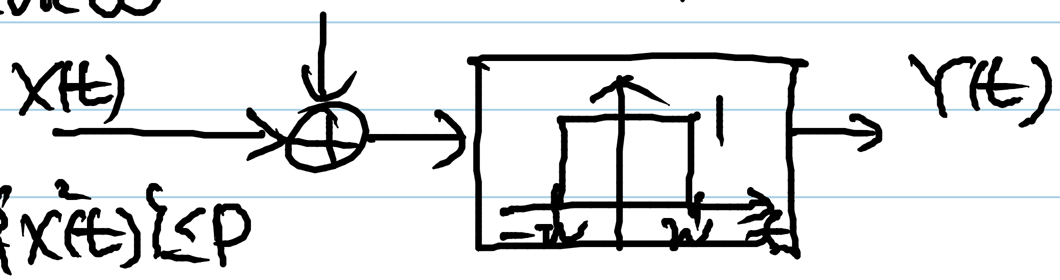
bit energy/noise density

ex) system A, system B, system C $\Rightarrow$ B > A > C

system D A $\gtrless$ D

system E, F

• Review $N(t)$ AWGN w/ $S_N(f)=\frac{N_0}{2}$



$E\{X^2(t)\}\le P$

Remind that

$$C = 2W\cdot\frac{1}{2}\log\left(1+\frac{P}{N_0W}\right)$$

[bps]
$$= W\log_2\left(1+\frac{P}{N_0W}\right)$$

Since

P [Watt] = [Joules/sec]

E_b [Joules/bit]

C [bits/sec]

$\Rightarrow P = E_b C$

we have

$$C = W\log_2\left(1+\frac{E_bC}{N_0W}\right)$$

implicit function

$$\Leftrightarrow \frac{C}{W} = \log_2\left(1+\frac{E_b}{N_0}\cdot\frac{C}{W}\right)$$

$$\Leftrightarrow 2^{\frac{C}{W}} = 1+\frac{E_b}{N_0}\cdot\frac{C}{W}$$

$$\Leftrightarrow \frac{2^{\frac{C}{W}}-1}{C} = \frac{E_b}{N_0}$$

inverse function

• Inverse function

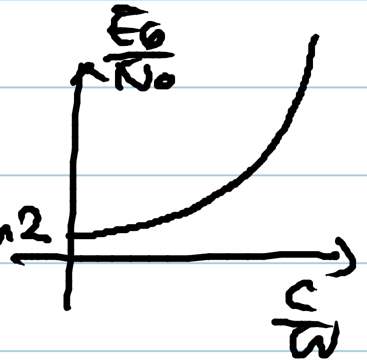
$$\frac{E_b}{N_o} = \frac{2^{\frac{C}{W}} - 1}{\frac{C}{W}}$$

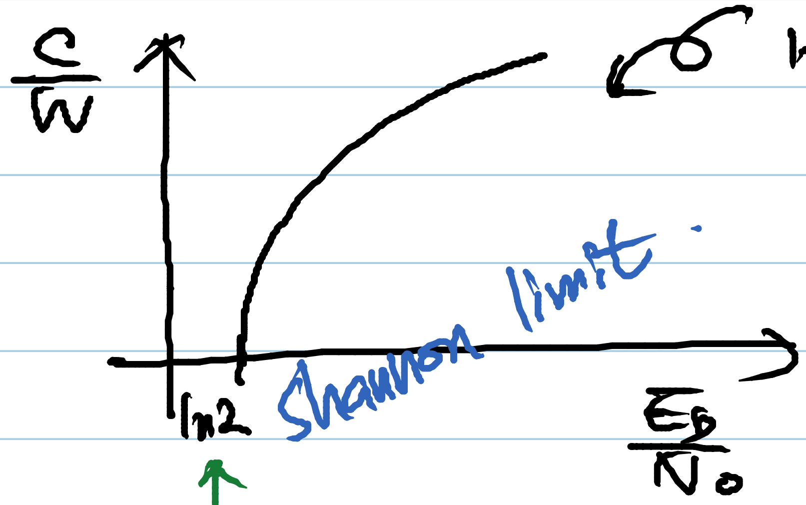
$\boxed{(1,1)}$

$$2^x = e^{\ln 2^x} = e^{x\ln 2}$$

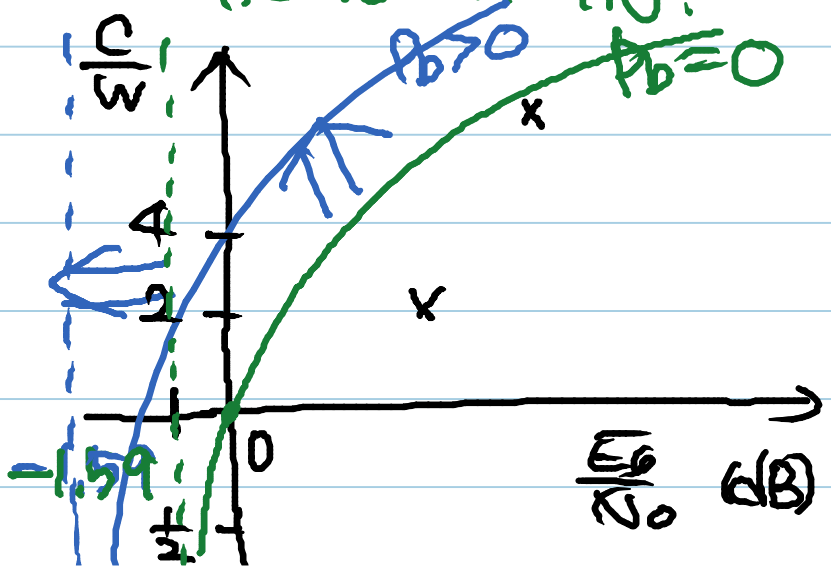
Consider $\frac{E_b}{N_o} > 0$, $\lim_{x \downarrow 0} \frac{2^x - 1}{x}$

$$= \lim_{x \downarrow 0} \frac{\ln 2 \cdot e^{x\ln 2}}{1} = \ln 2$$



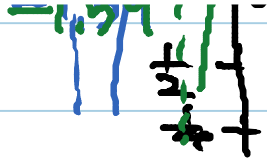


minimin repuired E_b/N_o for error-free Tx over AWGN.



Sklar

$$\frac{E_b}{N_o}\ (dB) = 10 \log_{10} \frac{E_b}{N_o}$$



$\overline{N_0}$ (dB) $= 10 \log_{10} \overline{N_0}$

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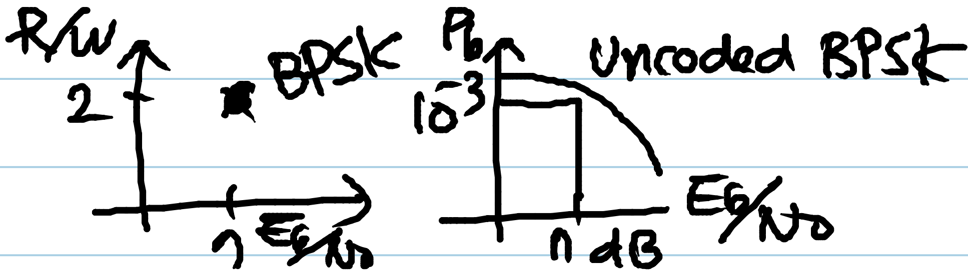


• Spectral Efficiency Plane w/ $P_b > 0$.

($P \leftarrow$ power)

$E_b \leftarrow$ bit energy

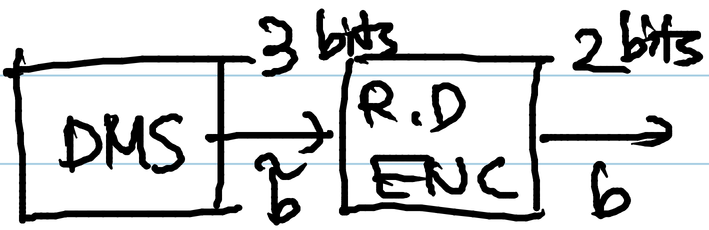
$C \leftarrow$ channel capacity

$N_0 \leftarrow$ noise density

$W \leftarrow$ band width

$$C = W \log\left(1 + \frac{E_b C}{N_0 W}\right)$$

(i)

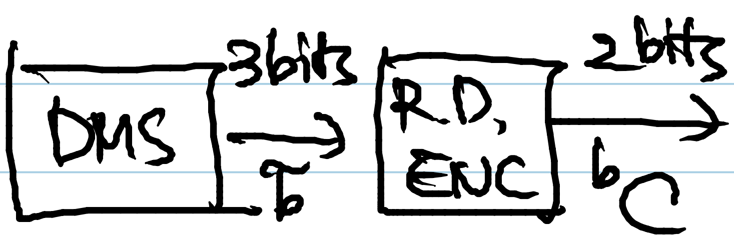


$$3\tilde{E}_b = 2E_b \Rightarrow \tilde{E}_b = \frac{2}{3}E_b$$

Generalize 하면

$$\tilde{E}_b = E_b\,(1 - h_b(P_b))$$

(ii)



$$\tilde{C} = \frac{3}{2}C.$$

$C = \frac{\;}{2} C$.

Generalize it!

$$\tilde{C} = \frac{C}{1-h_b(P_b)}$$

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오후 3:11

(i) & (ii) $\Rightarrow$

$$\frac{C}{W} = \log_2\left(1 + \frac{E_b}{N_0}\frac{C}{W}\right)$$

$$\Leftrightarrow \quad \frac{\tilde{C}(1-h_b(P_b))}{W} = \log_2\left(1 + \frac{\tilde{E}_b}{N_0}\frac{\tilde{C}}{W}\right)$$

$$\Leftrightarrow \quad \boxed{\frac{2^{\frac{\tilde{C}}{W}(1-h_b(P_b))} - 1}{\frac{\tilde{C}}{W}} = \frac{\tilde{E}_b}{N_0}}$$

Now,

$$\lim_{\frac{\tilde{C}}{W}\to 0} \frac{\tilde{E}_b}{N_0} = (1-h_b(P_b))\ln 2$$