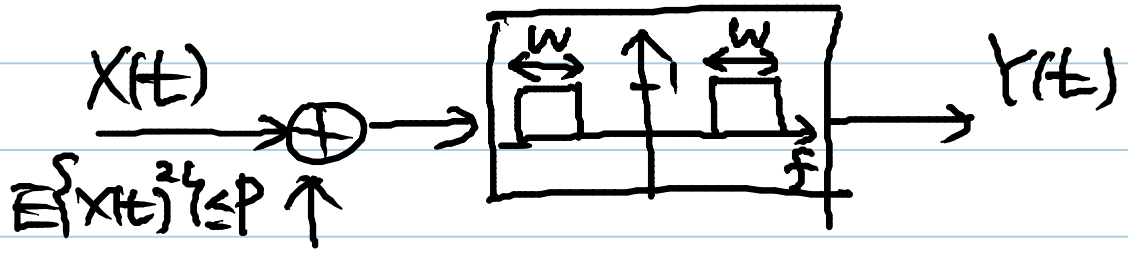
Lec. 33

2013년 6월 4일 화요일

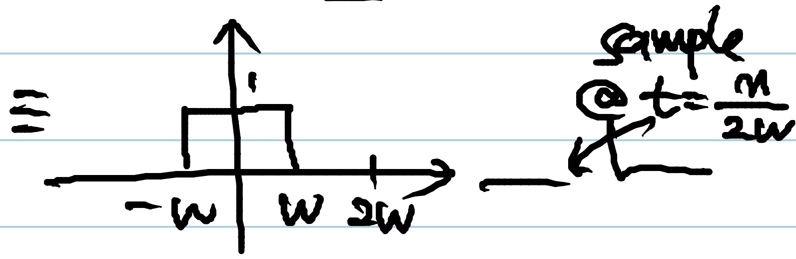
오후 1:50

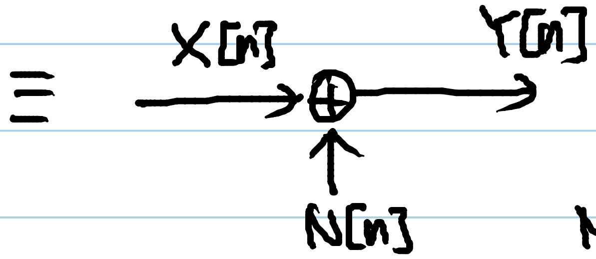
Review

– CT bandlimited AWGN channel



$S_{NN}(f) = \frac{N_0}{2}$





$N[n]$'s independent &

$N[n] \sim \mathcal{N}(0, N_0W)$

$E\{X[n]^2\} \le P$

$\int_{-\infty}^{\infty} S_{NN}(f) \underbrace{|H(f)|^2}_{\text{power transfer function}} df = N_0W$

$r = \frac{k}{n}$ ← bit ← channel use

$r = \frac{1}{2}\log_2\left(1 + \frac{P}{N_0W}\right)$ [bits/channel use]

$2Wr = C$ [bits/dimension]

W Hz → 2W channel use/sec $P = E_bC$

$C = W\log_2\left(1 + \frac{P}{N_0W}\right)$ [bits/sec]

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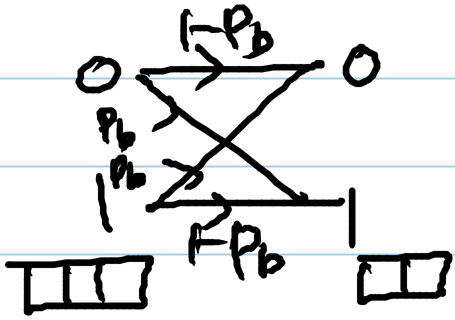
- w/ distortion



$p_{\hat{X}|X}(\hat{x}|x)$

$R(P_b) = 1 - h_b(P_b) \le 1$

$\tilde{E}_b = E_b(1-h_b(P_b))$

$$\tilde{C} = \frac{C}{1-h_b(P_b)}$$

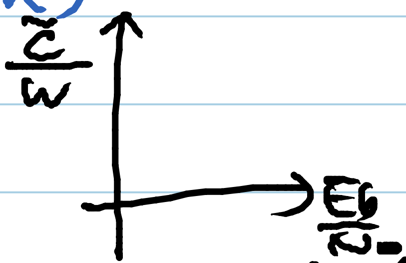
$$\tilde{C}(1-h_b(P_b)) = W\log_2\left(1+\left(\frac{\tilde{E}_b}{N_0}\right)\left(\frac{\tilde{C}}{W}\right)\right)$$

- Spectral Efficiency (Bandwidth)

[bits/sec/Hz]

= [bps/Hz]

$= \frac{\tilde{C}}{W}$



$$\frac{\tilde{C}}{W}(1-h_b(P_b)) = \log_2\left(1+\left(\frac{\tilde{E}_b}{N_0}\right)\left(\frac{\tilde{C}}{W}\right)\right)$$

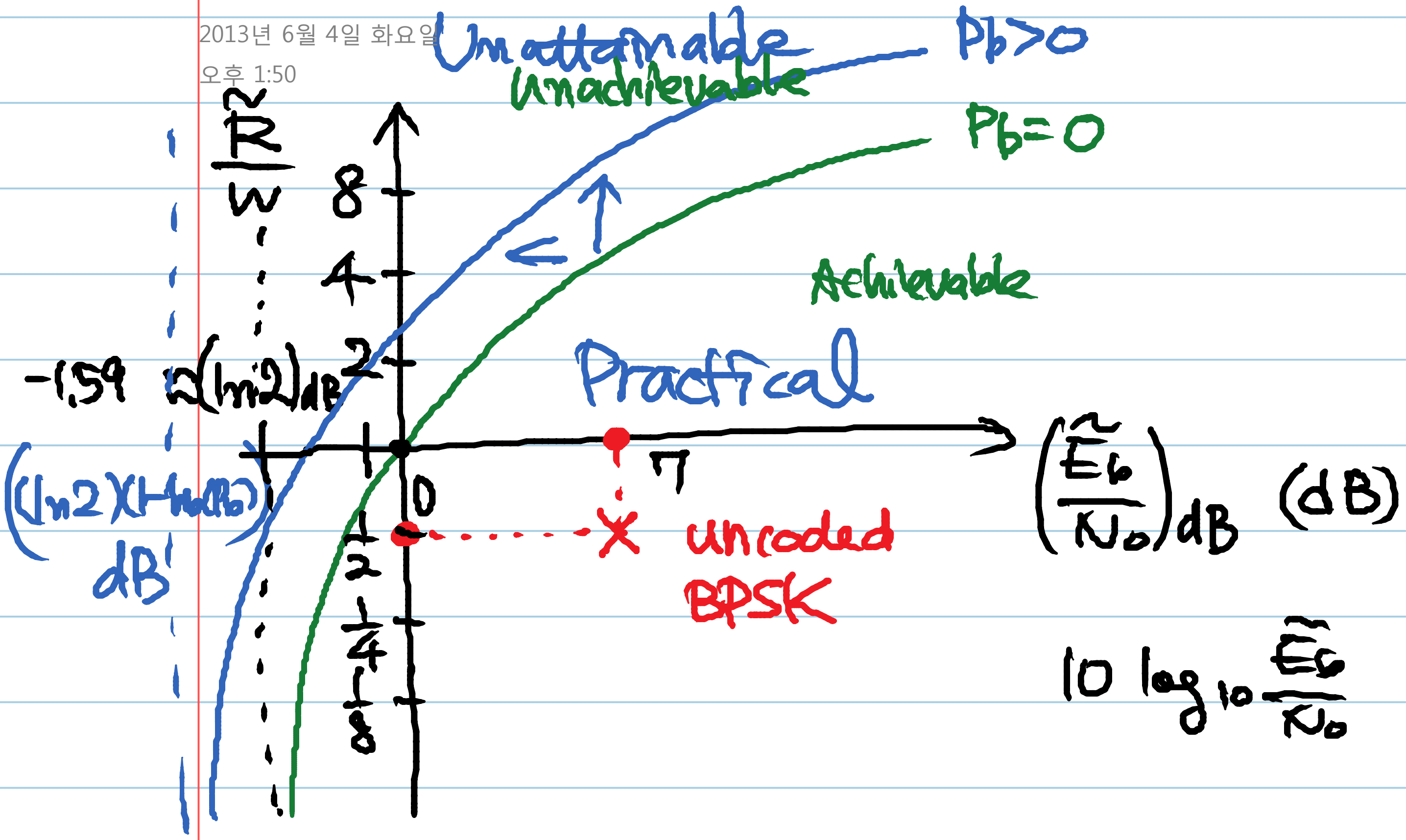
$$\frac{\tilde{E}_b}{N_0} = \frac{2^{\frac{\tilde{C}}{W}(1-h_b(P_b))}-1}{\frac{\tilde{C}}{W}}$$

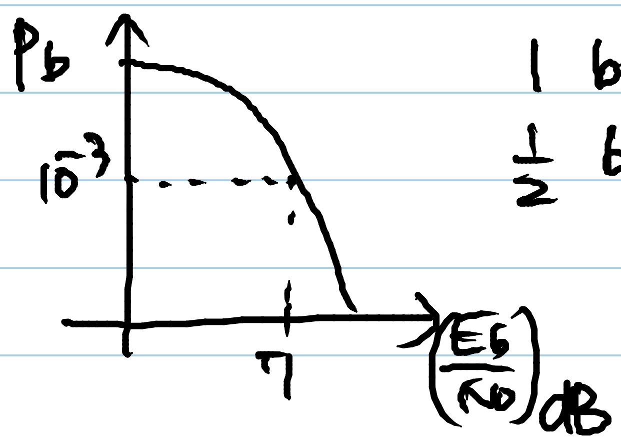
$y=f(x)$

$y=af(bx)$

$$\lim \frac{\tilde{E}_b}{N_0} = (\ln 2)(1-h_b(P_b))$$

좋 → 0 No



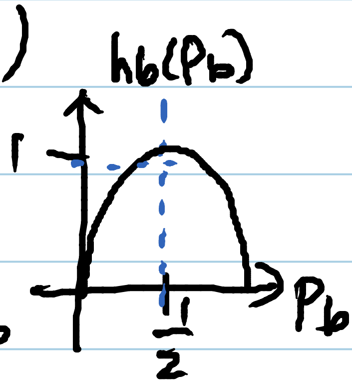


1 bit /channel use

$\frac{1}{2}$ bits/Hz

- P_b as a function of $\frac{\tilde{C}}{W}$ & $\frac{\tilde{E}_b}{N_o}$

$$\frac{\tilde{C}}{W}(1-h_b(P_b)) = \log_2\left(1+\left(\frac{\tilde{E}_b}{N_o}\right)\left(\frac{\tilde{C}}{W}\right)\right)$$



Define h_b^{-1} as the inverse ft of h_b

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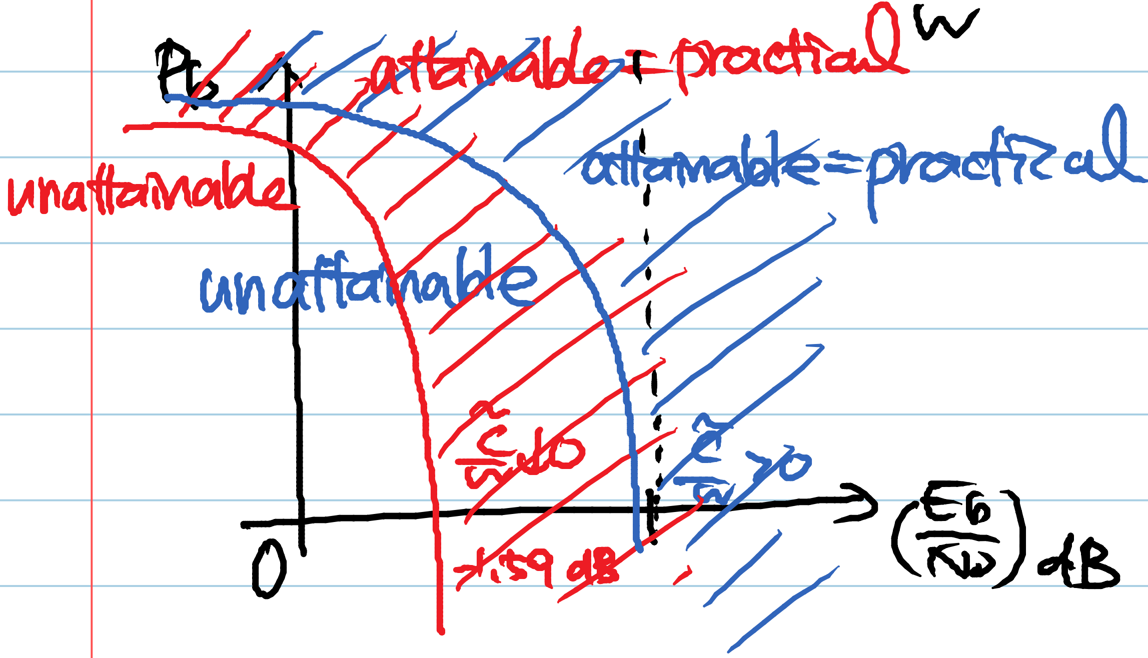
$$P_b = h_b^{-1}\left(1 - \frac{\log_2\left(1 + \left(\frac{\tilde{E}_b}{N_0}\right)\left(\frac{\tilde{C}}{W}\right)\right)}{\frac{\tilde{C}}{W}}\right)$$

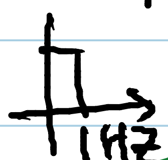


- $r = \frac{k}{n}$ [bits/channel use]

$\tilde{r}$ $(P_b > 0)$

$\frac{\tilde{C}}{W}$ [bits/sec/Hz]

1 [Hz] = 2 [channel use/sec]

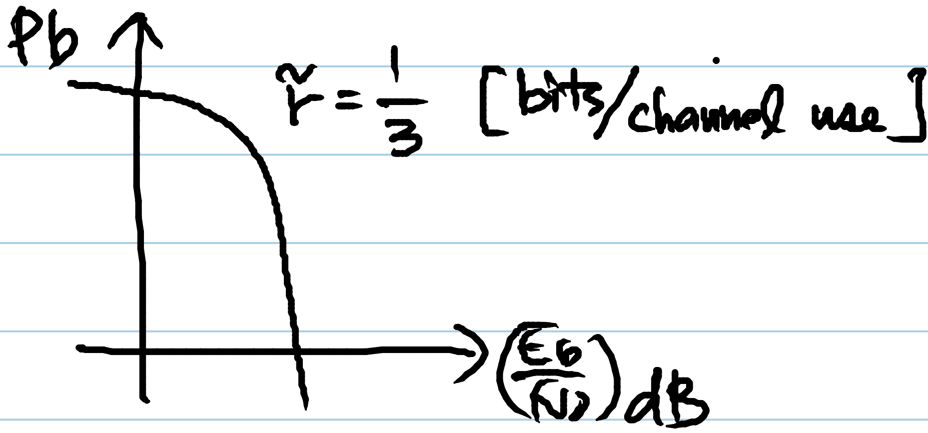


1/channel use = 2/sec/Hz

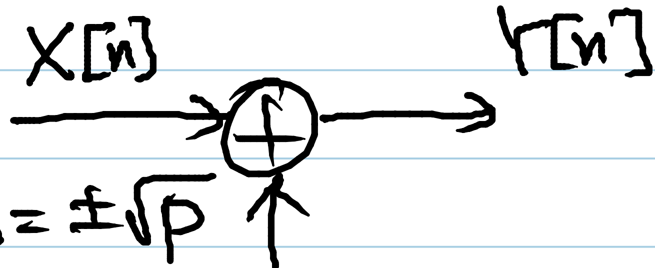
$$\frac{\tilde{C}}{W} = 2\tilde{r}$$

$$2\tilde{r}\left(1-h_b(P_b)\right)=\log_2\left(1+\frac{2\tilde{E}_b}{N_0}\tilde{r}\right)$$

$$\Rightarrow P_b = h_b^{-1}\left(1-\frac{\log_2\left(1+\frac{2\tilde{E}_b}{N_0}\cdot\tilde{r}\right)}{2\tilde{r}}\right)$$



- Constellation - Restricted



$X[n] = \pm\sqrt{P}$

binary antipodal signaling

N[n]'s i.i.d. $\sim N(0, \sigma^2)$

r [bits/channel use]

$r < 1$

$$r = \max_{x=\pm\sqrt{P}} I(X;Y)$$

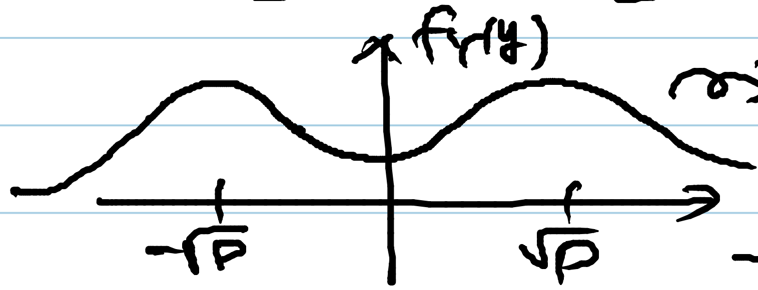
$$I(X;Y) = h(Y) - h(Y|X) = H(X) - H(X|Y)$$

$$Y = X + N$$

$$N \sim N(0,\sigma^2)$$

$$f_X(x) = \frac{1}{2}\delta(x-\sqrt{P}) + \frac{1}{2}\delta(x+\sqrt{P})$$

$$f_Y(y) =$$



$\rightsquigarrow h(Y) - \frac{1}{2}\log 2\pi e\sigma^2$

We have

$$r\left(\frac{P}{\sigma^2}\right). \quad \text{[bits/channel use]}$$

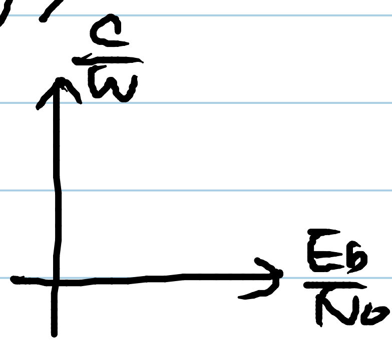
- CT:

$$C = 2W\, r\left(\frac{P}{\sigma^2}\right) \text{ [bits/sec]}$$

$$P = E_b C$$
$$\sigma^2 = N_0 W$$

$$\frac{C}{W} = 2\, r\left(\frac{E_b}{N_0}\frac{C}{W}\right)$$

$$\frac{r^{-1}\left(\frac{1}{2}\frac{C}{W}\right)}{\frac{C}{W}} = \frac{E_b}{N_0}$$



- $P_b > 0$

$$\tilde{C}(1-h_b(P_b)) = 2W\, r\left(\frac{\tilde{E}_b}{N_0}\frac{\tilde{C}}{W}\right)$$

$$P_b = h_b^{-1}\left(1 - 2r\left(\frac{E_b}{N_0}\frac{C}{W}\right)\right).$$

$$p_b = n_b^{-1}\left(1 - \frac{2r\left(\frac{E_b}{\sigma_b} - \frac{\sigma}{\sigma_w}\right)}{\frac{\sigma}{\sigma_w}}\right).$$

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$$P_b = \overline{n_b}\left(1 - \frac{\cancel{2}\, r\left(\frac{2\tilde{E}_b}{N_0}\tilde{r}\right)}{\cancel{2}\,\tilde{r}}\right)$$