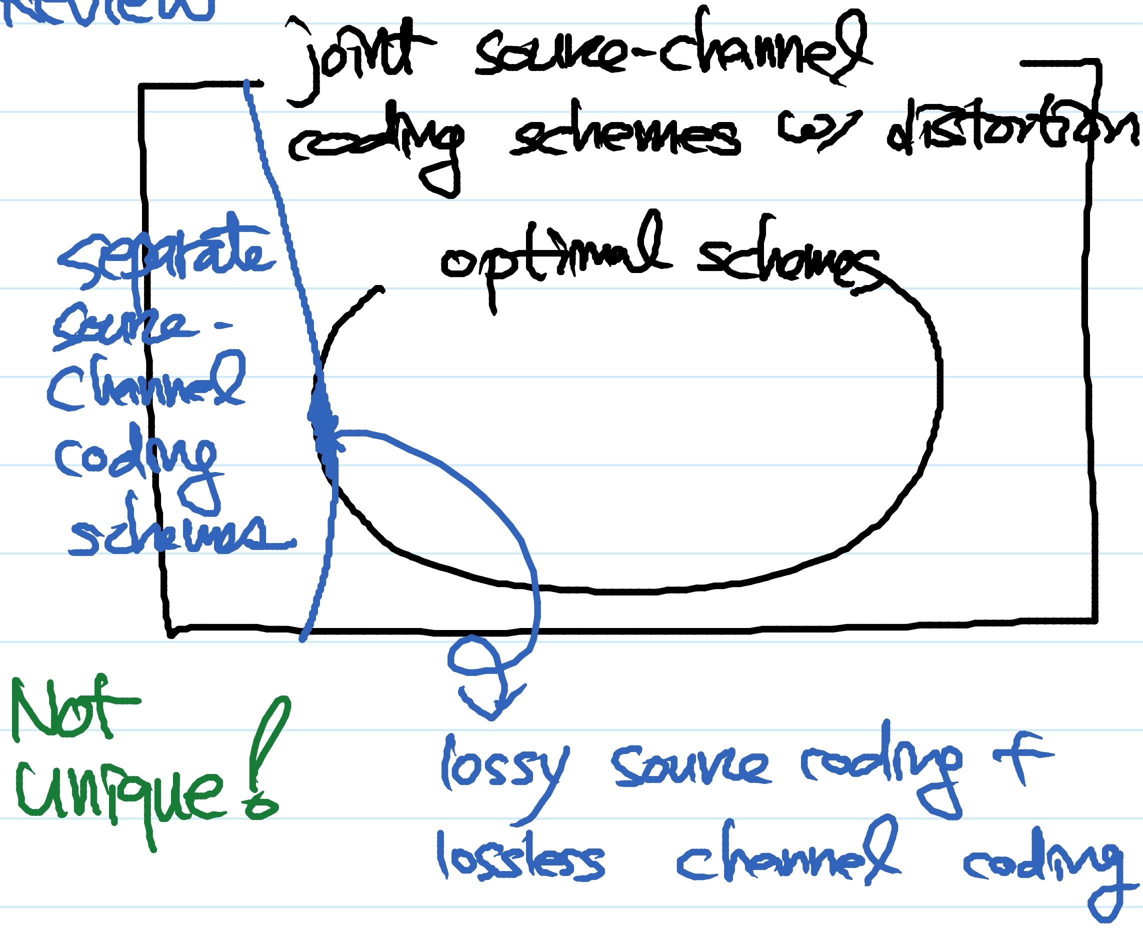
Lec. 34

2013년 6월 4일 화요일

오후 8:57

- Review



/ex/ linear modulation w/ negative excess bandwidth

≡ faster-than-Nyquist { transmission / modulation }

⇒ ISI FTN

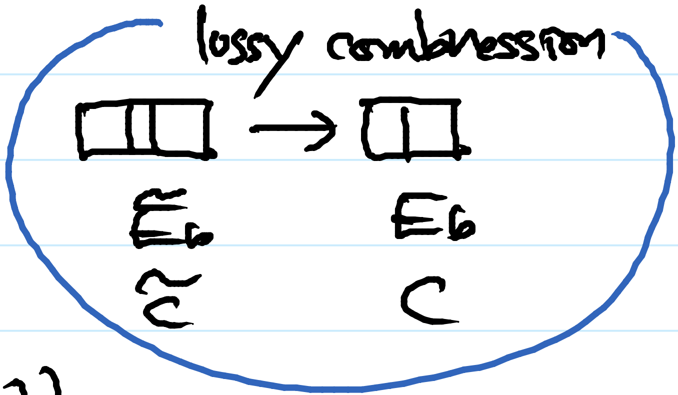
≡ super Nyquist transmission

≡ ...

- $C = W\log_2\left(1+\frac{P}{N_0W}\right)$ [bits/sec]

$P = E_bC$

$\tilde{C} = \frac{C}{1-h_b(P_b)}$

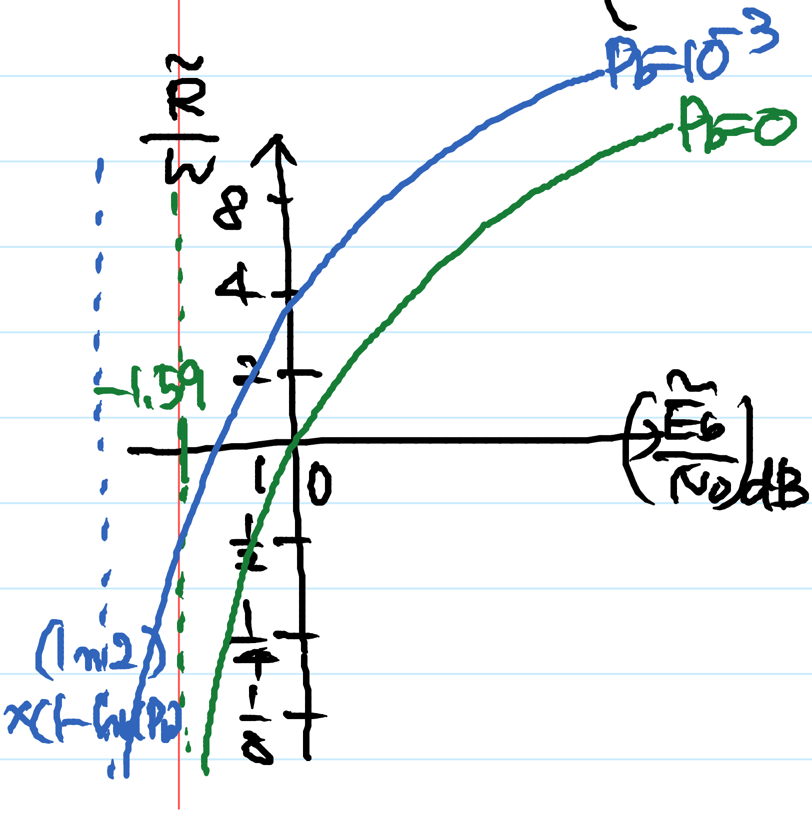


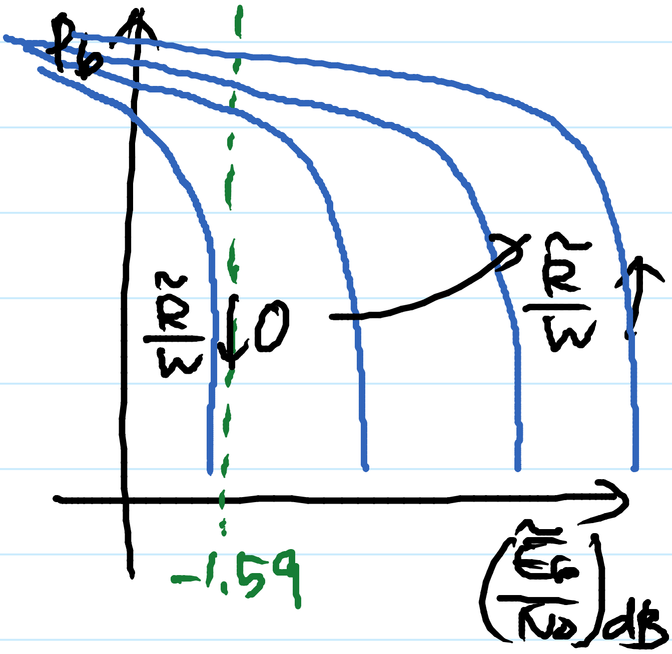
$\tilde{E}_b = E_b(1-h_b(P_b))$

implicit function

(i) $$\frac{\tilde{E}_b}{N_0} = \frac{2^{\frac{\tilde{C}}{W}(1-h_b(P_b))}-1}{\frac{\tilde{C}}{W}}$$

(ii) $$P_b = h_b^{-1}\left(1-\frac{\log_2\left(1+\left(\frac{\tilde{E}_b}{N_0}\right)\left(\frac{\tilde{C}}{W}\right)\right)}{\frac{\tilde{C}}{W}}\right)$$



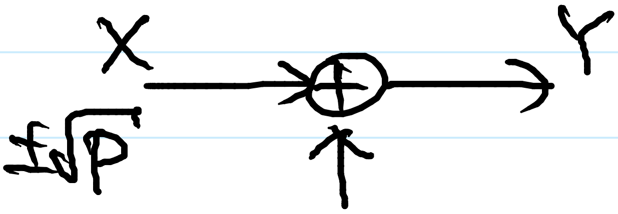


$$r = \frac{k}{n} \quad [\text{bits/channel use}]$$

$$\tilde{r}$$

$$\boxed{\frac{\tilde{C}}{W} = 2\tilde{r}} \Rightarrow P_b = h_b^{-1}\left(1 - \frac{\log_2\left(1+\frac{2\tilde{E}_b}{N_0} r\right)}{2\tilde{r}}\right)$$

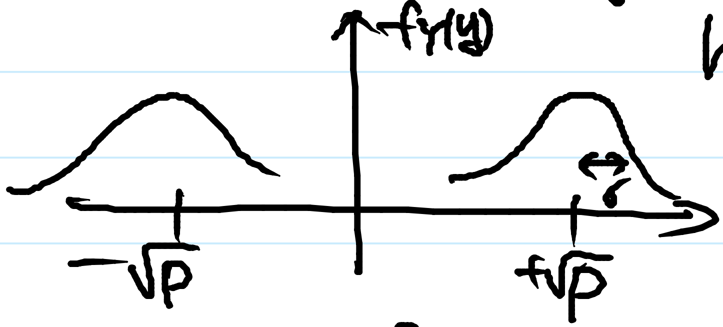
- BPSK constellation



N i.i.d. $\sim N(0, \sigma^2)$

$$I(X;Y) = h(Y) - \frac{1}{2}\log(2\pi e\sigma^2)$$

$$h(Y) = E\left\{\log\frac{1}{f_Y(Y)}\right\}$$



$$= r\left(\frac{P}{\sigma^2}\right)$$

- CT

$$C = 2W\, r\left(\frac{P}{N_0 W}\right)$$

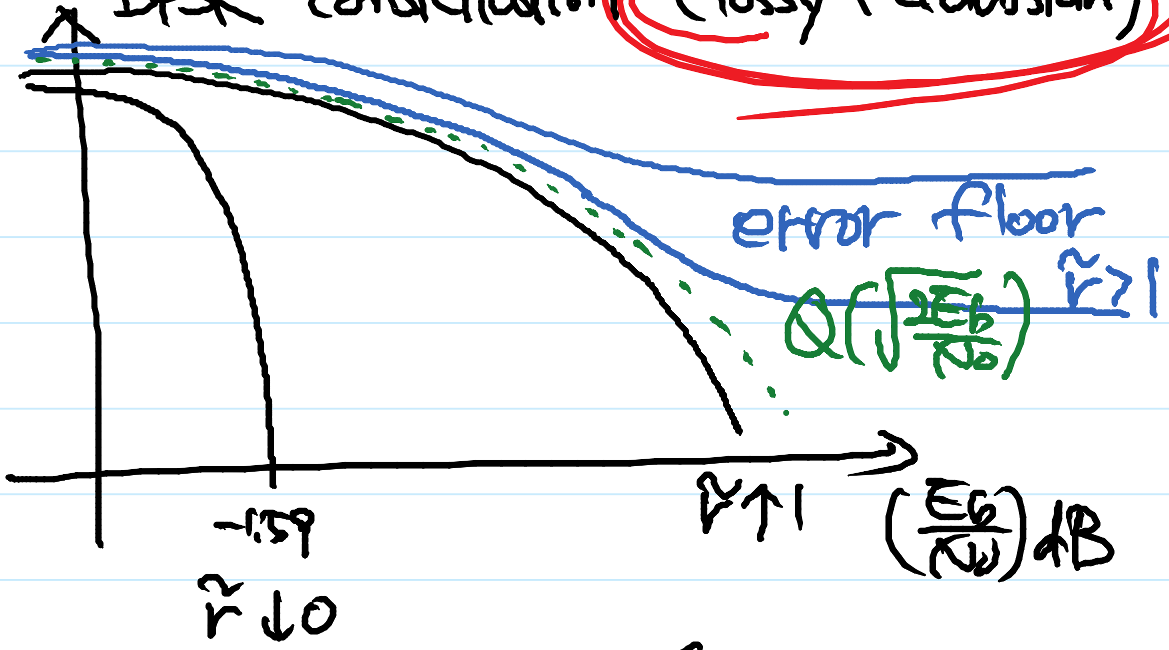
$$\frac{\tilde{C}}{W}\left(1 - h_b(P_b)\right) = 2r\left(\frac{\tilde{E}_b}{N_0} \cdot \frac{\tilde{C}}{W}\right)$$

$$P_b = \tilde{h}_b^{-1}\left(1 - \frac{2r\left(\frac{\tilde{E}_b}{N_0}\cdot\frac{\tilde{C}}{W}\right)}{\frac{\tilde{C}}{W}}\right)$$

$$\frac{\tilde{C}}{W} = 2\tilde{r}$$

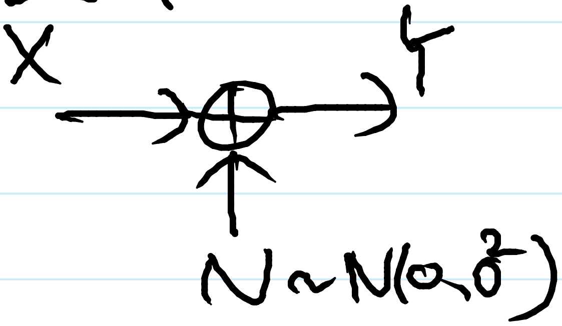
$$\Rightarrow P_b = \tilde{h}_b^{-1}\left(1 - \frac{r\left(\frac{2\tilde{E}_b}{N_0}\cdot\tilde{r}\right)}{\tilde{r}}\right)$$

BPSK constellation (lossy + Gaussian)

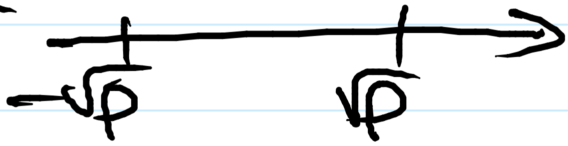


$$E[X^2] \le P$$

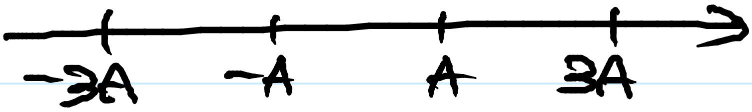
- PAM $\underbrace{\text{constellation}}_{\neq \text{ modulation}}$



BPSK



4-PAM



Uniform distribution

$E[X^2] \le P$

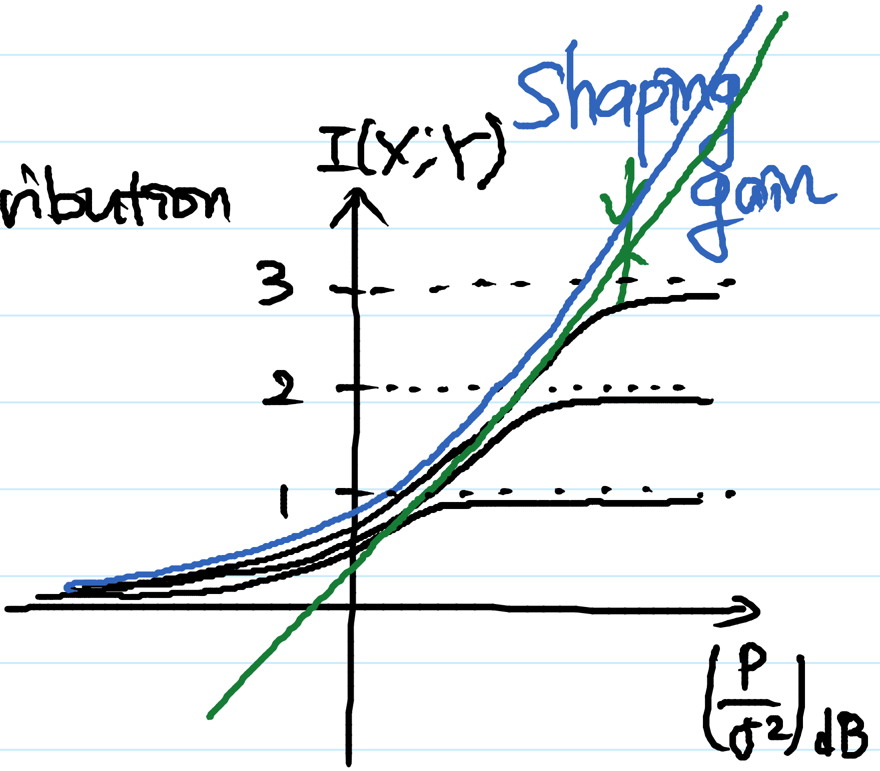
$N \sim \mathcal{N}(0, \sigma^2)$

2-PAM

4-PAM

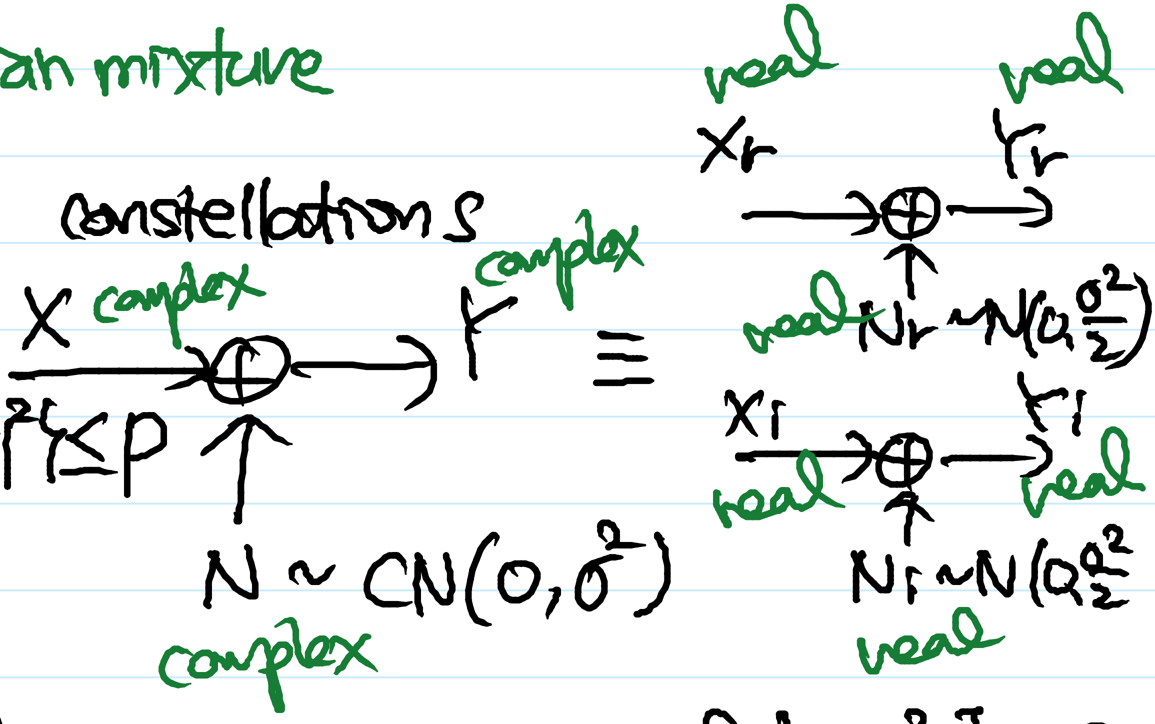
8-PAM

⋮



Gaussian mixture

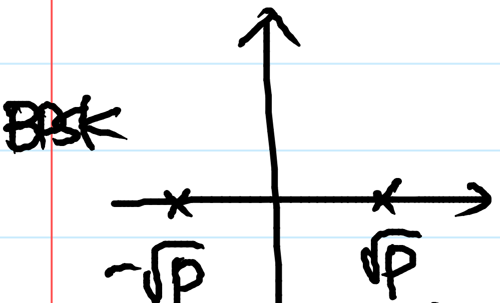
- QAM constellations

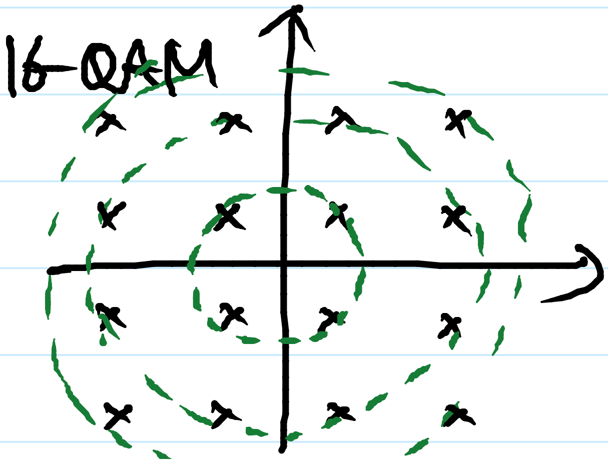


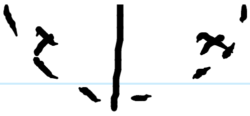
$E\{|X|^2\} \le P$

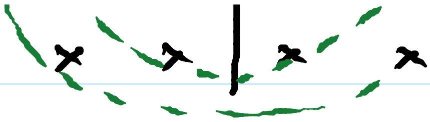
$N \sim \mathcal{CN}(0, \sigma^2)$ complex

$E[X_r^2 + X_i^2] \le P$









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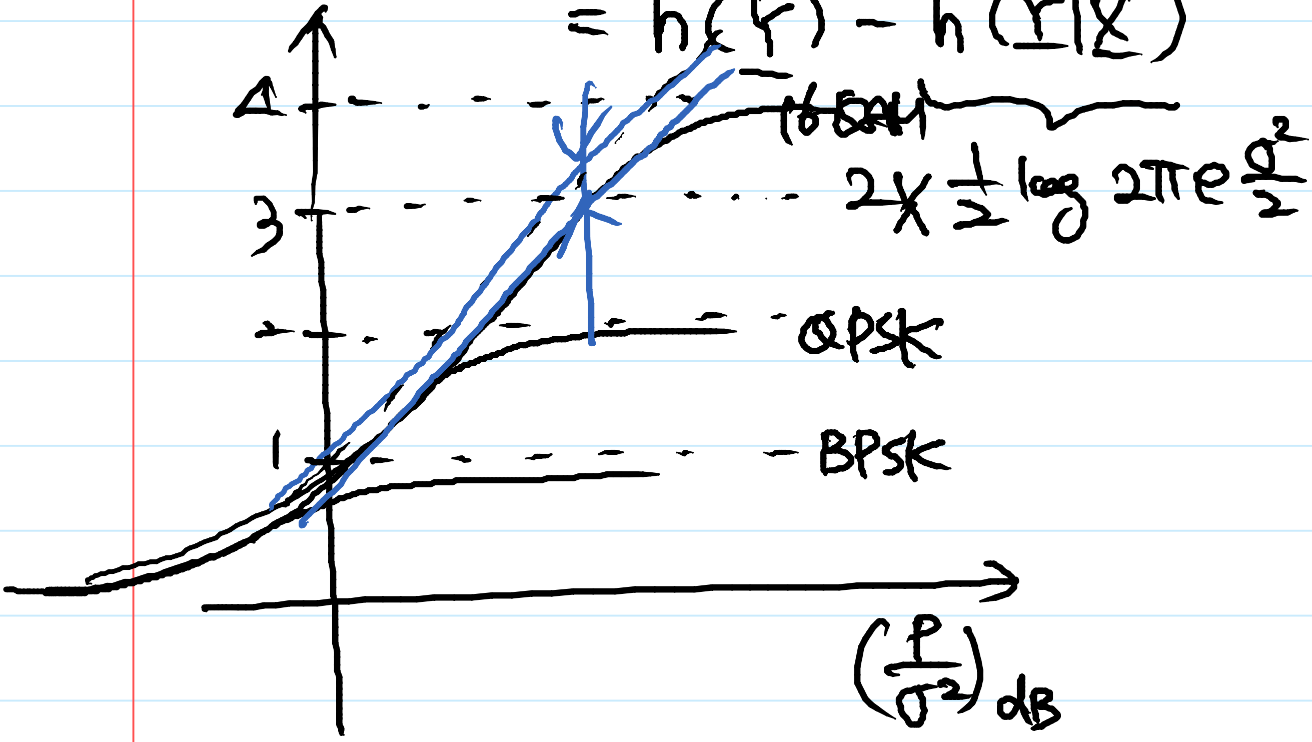
$$I(X;Y) = I\left(\begin{bmatrix}X_r\\X_i\end{bmatrix};\begin{bmatrix}Y_r\\Y_i\end{bmatrix}\right)$$

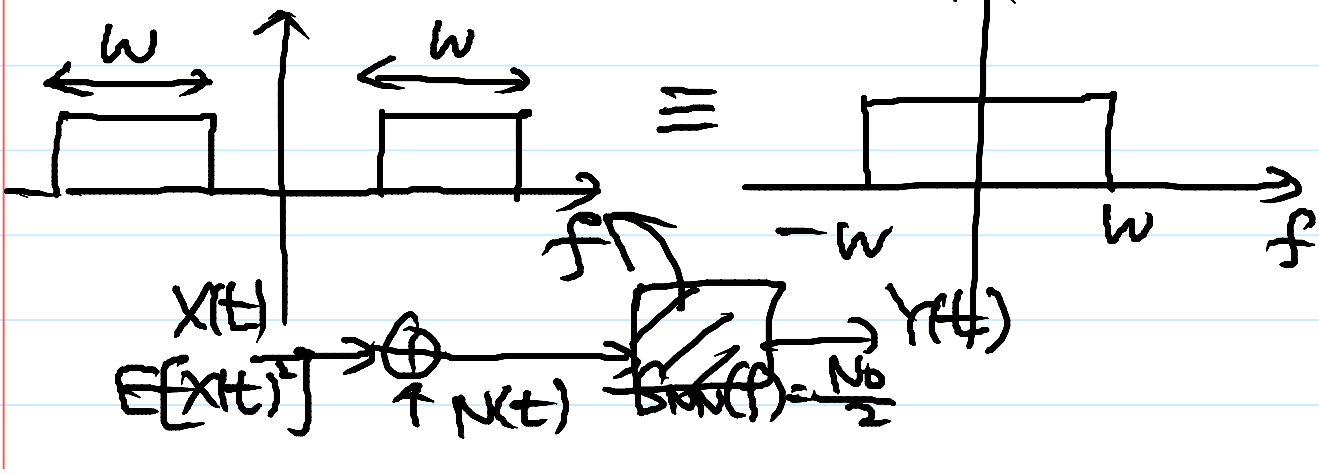
$$= I(X_r, X_i ; Y_r, Y_i)$$

$$= E\left\{\log \frac{f_{X_r,X_i,Y_r,Y_i}(X_r,X_i,Y_r,Y_i)}{f_{X_r,X_i}(X_r,X_i)\, f_{Y_r,Y_i}(Y_r,Y_i)}\right\}$$

$$= h(Y) - h(\underline{Y}|\underline{X})$$



- QAM in CT.



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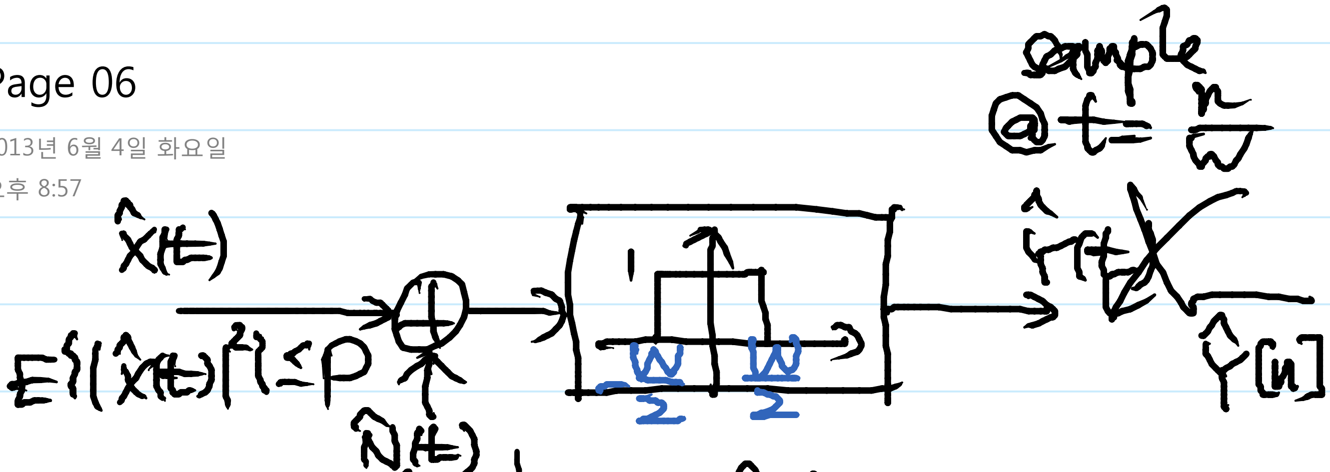
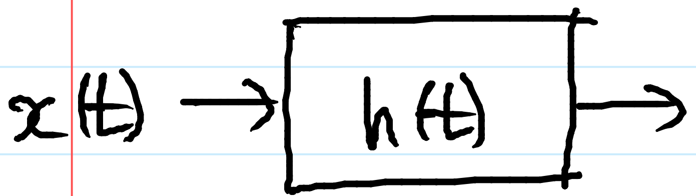


$E\{|\hat{x}(t)|^2\} \le P$

$x(t) = \mathrm{Re}\{x_\ell(t)\, e^{j2\pi f_c t}\}$

$S_{\hat{N}\hat{N}}(f) = N_0$



$y(t) = h(t) * x(t)$

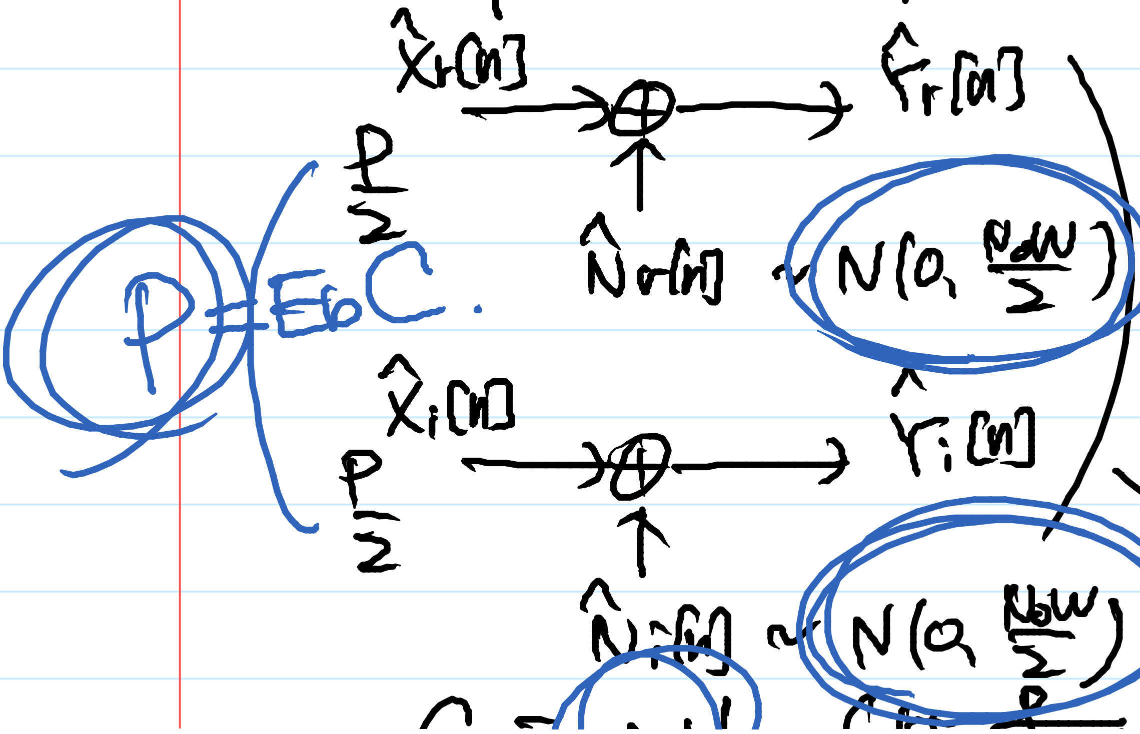
$x(t) \triangleq \mathrm{Re}\{x_\ell(t)\, e^{j2\pi f_c t}\}$

$h(t) \triangleq \mathrm{Re}\{h_\ell(t)\, e^{j2\pi f_c t}\}$

$y(t) \triangleq \mathrm{Re}\{y_\ell(t)\, e^{j2\pi f_c t}\}$

$y_\ell(t) = \frac{1}{2} h_\ell(t) * x_\ell(t)$

W [complex channels/sec]



$P = E_b C.$

$\hat{N}_r[n] \sim N(0, \frac{N_0 W}{2})$

$\hat{N}_i[n] \sim N(0, \frac{N_0 W}{2})$

$\frac{1}{2}\log(1+\frac{\frac{P}{2}}{\frac{N_0 W}{2}})$

$= \frac{1}{2}\log(1+\frac{P}{N_0 W})$

$\log(1+\frac{P}{N_0 W})$

$$C = W \log\left(1 + \frac{P}{N_0 W}\right).$$

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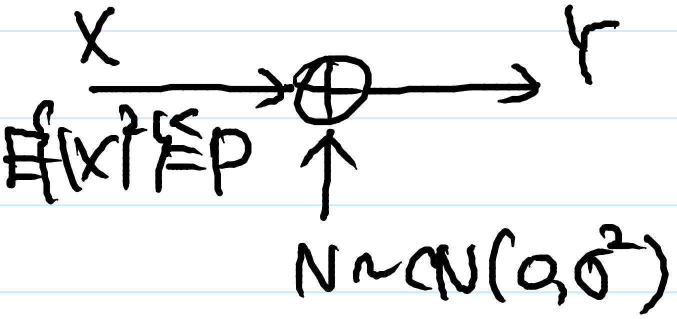
오후 10:07

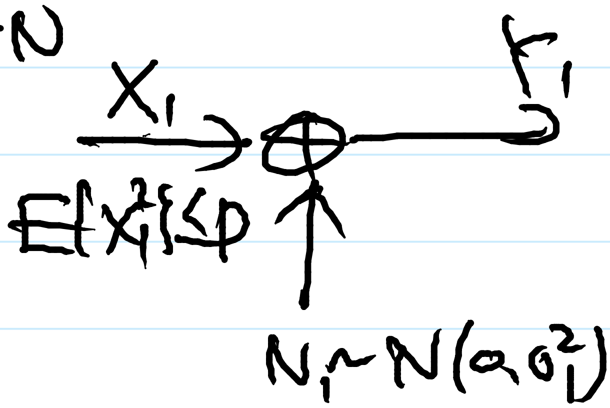
- Back to QAM constellations



$\sigma^2 = N_0 W$.

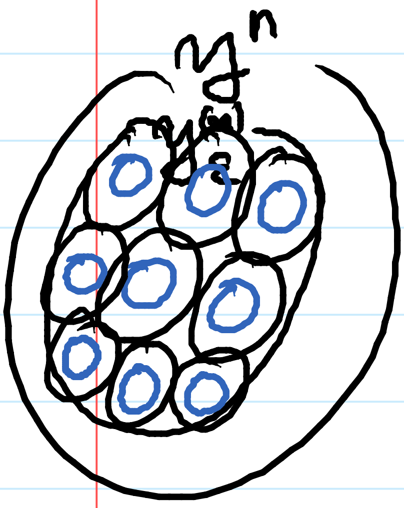
- Gaussian codebook.

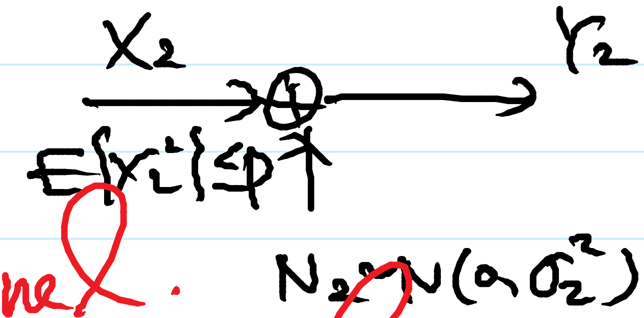
DT-AWGN



$C_1 = \frac{1}{2}\log(1+\frac{P}{\sigma_1^2})$

$\mathcal{C}_1$





$\sigma_1^2 < \sigma_2^2$

$\Rightarrow C_1 > C_2$.

$C_2 = \frac{1}{2}\log(1+\frac{P}{\sigma_2^2})$

$\mathcal{C}_2$

Channel.

Outage!

Q1. What if you use $\mathcal{C}_1$ for channel 2?

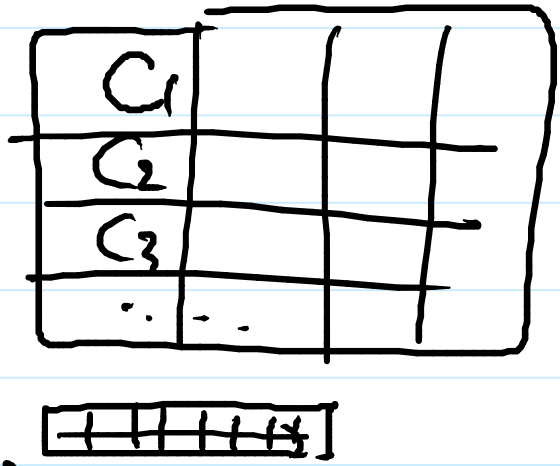
Decoding failure w/ R=0.

Q2. " " " " $\mathcal{C}_2$ for " 1?

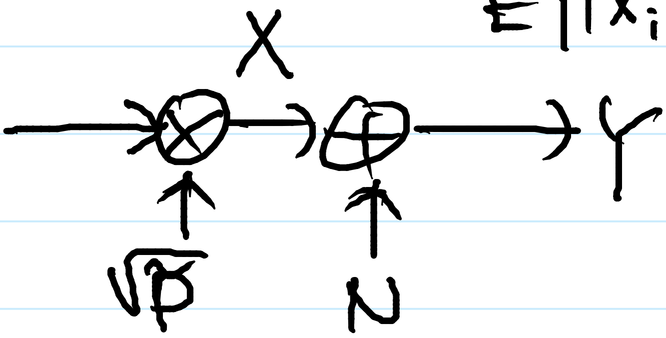
Decoding success w/ $R = 6_2$

Q3. $\tilde{P}, \tilde{\sigma}^2$

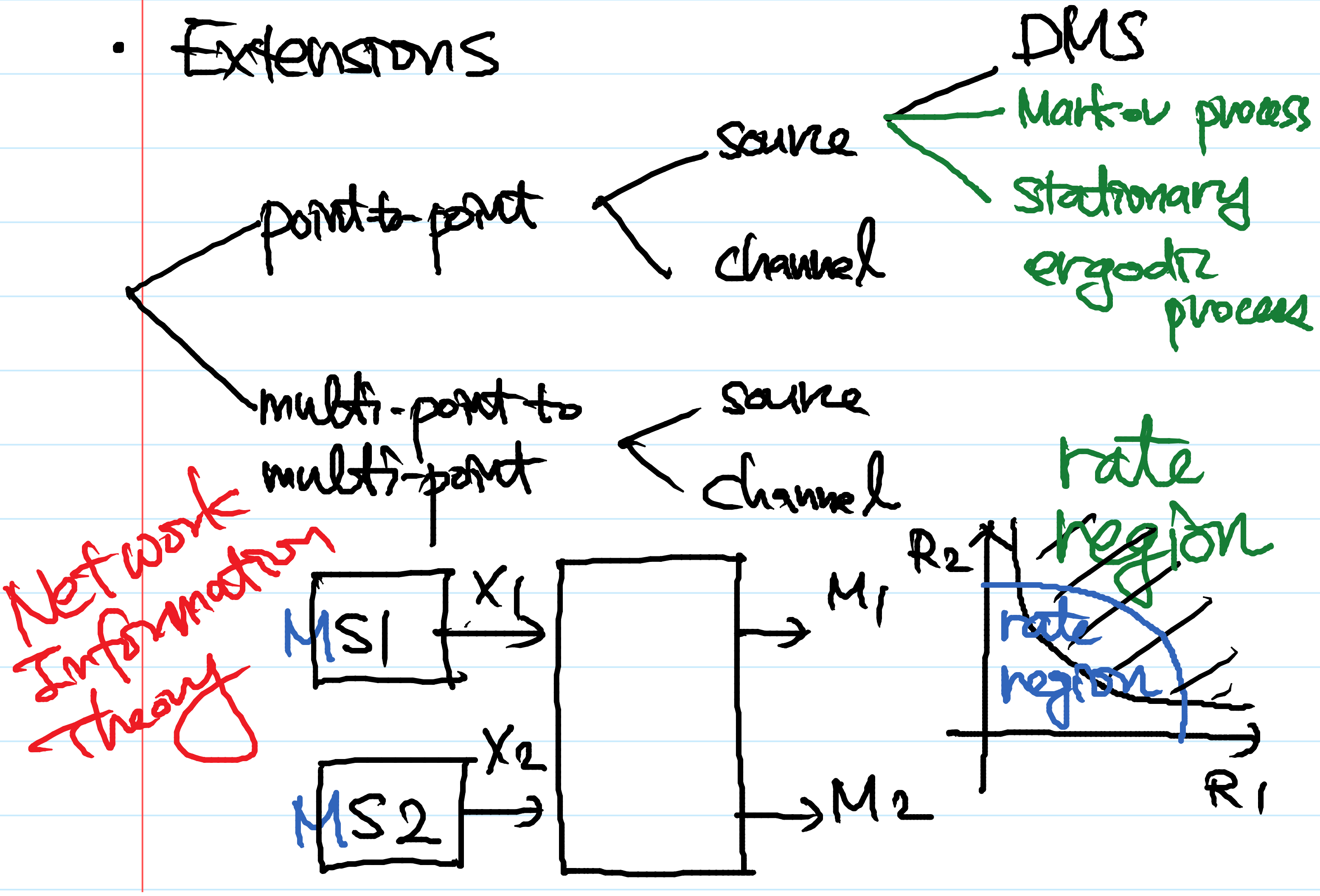
$$\tilde{C} = \frac{1}{2}\log\left(1+\frac{\tilde{P}}{\tilde{\sigma}^2}\right)$$



$E\{|x_i^{(m)}|^2\} = 1, \forall i, n,$



- Extensions



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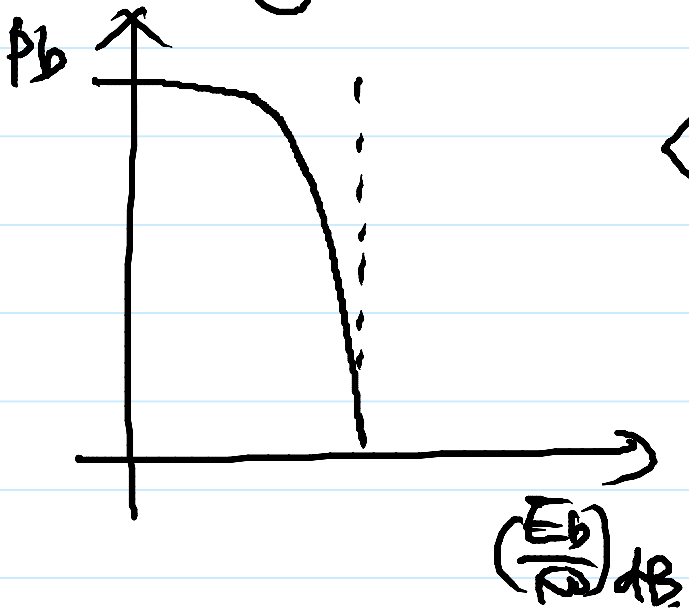
오후 10:30

Summary



< Visualization !
Simplification !