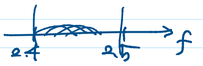
220427 Channel Capacity

2022년 4월 27일 수요일 오후 4:09

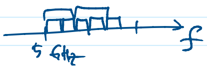
□ Channel

< academy :
industry : band /ex/ 2.4 GHz ISM bands

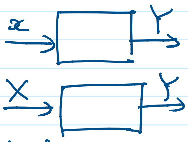


channel

subchannel



□ Memoryless Channel



$f_{Y;x}(y)$

$f_{Y|X}(y|x)$

$\{X=x\}$

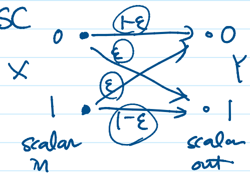
admissible input

/ex/ $x \in \{0,1\}$

$x \in \mathbb{R}$

$\{f_{Y|X}(y|x) : x \in \mathbb{R}\}$

□ BSC



$0 \le \varepsilon \le 1.$

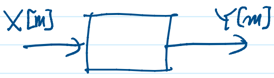
$\{X=x\}$ $x \in \{0,1\}$

$\{Y=y\}$ $y \in \{0,1\}$

Q. How much information?

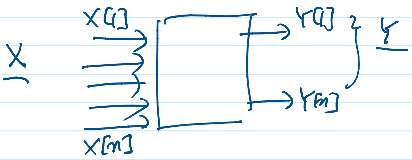
A. (transmission rate / error rate) bits/channel use $\frac{K}{n}$ bits/channel use

□ Memoryless Channel



$m = 1, 2, \cdots, n.$

< CMC (CICOMC)
DMC (DIDOMC)



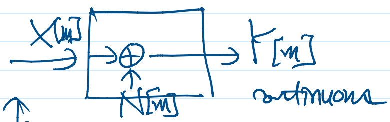
$\underline{X}$ $\underline{Y}$

$$f_{\underline{Y}|\underline{X}}(\underline{y}|\underline{x})$$

$$= \prod_{m=1}^{n} f_{Y[m]|X[m]}\left(y[m] \,|\, x[m]\right)$$

$$\overset{\text{memoryless}}{=} \prod_{m=1}^{n} f_{Y|X}\left(y[m] \,|\, x[m]\right)$$

□ ex.



continuous

DICO-MC

16-APSK

discrete input

$N \sim CN(0, \sigma^2)$

□ Channel Capacity

Given a channel, there is the capacity of the channel

such that if $r = \frac{k}{n} < C$

then we can $P_b \simeq 0.$

then we can $P_b \simeq 0$.

□ ex. $X \rightarrow \oplus \rightarrow Y$ $\quad E\{X^2\} \le P$

$N \sim \mathcal{N}(0, \sigma^2)$ $\quad \rightarrow \quad C = \frac{1}{2}\log_2\left(1 + \frac{P}{\sigma^2}\right)$

AWGN ch.

DT AWGN channel $\qquad$ [bits/channel]

metrics

□ $H(X)$ the entropy of a discrete RV X

$H(X,Y)$ the joint entropy of two discrete RVs X & Y.

$I(X;Y)$ the mutual information of two discrete RVs.

$h(X)$ the differential entropy of a continuous RV X

$H(X|Y)$ the conditional " " " discrete RV X given Y

$h(X|Y)$ " " differential " " continuous RV X given Y

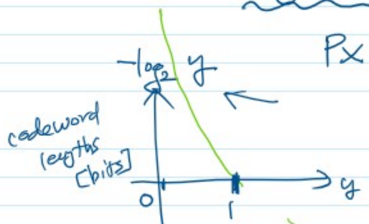
□ Def

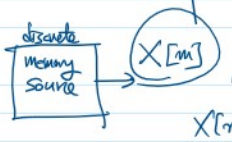
$$H(X) = E[-\log_2 p_X(X)] = \sum_{x \in \mathcal{X}} \left(-\log_2 p_X(x)\right) p_X(x)$$

$p_X(x) = g(x)$ $\quad E\left(-\log_2 g(X)\right)$

PMF of

길이 X 확률

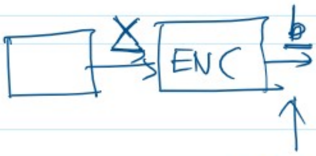




$m = 1, 2, \ldots$

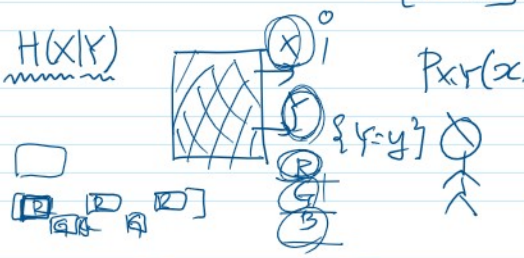
$X[m]$'s i.i.d.

$$\begin{bmatrix} X[1] \\ \vdots \\ X[10^6] \end{bmatrix} = \underline{X} \qquad \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$



$E[\ell(\underline{b})]$

□ $H(X|Y)$



$$p_{X,Y}(x,y) = p_Y(y)\, p_{X|Y}(x|y)$$

$$H(X,Y) = H(Z) = H(Y) + H(X|Y)$$

$\ln p_{X,Y} = p_Y p_{X|Y}$

$$H_{Y=y}(X) = E\left[-\log_2 p_{X|Y}(X|y) \,\middle|\, Y=y\right]$$

$$= \boxed{\sum_{x \in \mathcal{X}} \left(-\log_2 p_{X|Y}(x|y)\right) p_{X|Y}(x|y)}$$

$$\sum_{y \in \mathcal{Y}} H_{Y=y}(X)\, p_Y(y) = H(X|Y)$$

$p_{X,Y}(x,y)$

$$= \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} \left(-\log_2 p_{X|Y}(x|y)\right) \boxed{p_{X|Y}(x|y)\, p_Y(y)}$$

[illegible]

$$= \underline{E}\left[-\log_2 P_{X|Y}(X|Y)\right]$$

$P_{X|Y}(x,y)$ $\{Y=y\}$ $\{X=x\}$

☐ BSource

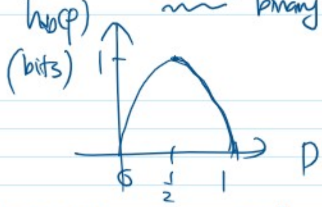
X $x\in\{0,1\}$

$P(X=0)=p.$ $P(X=1)=1-p$

$H(X)$

$h_b(p) = -(\log_2 p)p - (\log_2(1-p))(1-p)$

binary entropy ft



☐ $I(X;Y)$ mutual information.

$$P_{X,Y}(x,y) = P_X(x)P_{Y|X}(y|x) = P_Y(y)P_{X|Y}(x|y)$$

$$H(X,Y) = H(X) + H(Y|X) = H(Y) + H(X|Y)$$

$$\Rightarrow H(X) - H(X|Y) = H(Y) - H(Y|X) \triangleq I(X;Y)$$

$$= I(Y;X)$$

X Y

Y를 observe 했을 때 resolve 된 X의 uncertainty

(uncertainty reduction 量)

☐ Channel

$x \rightarrow$ channel $\rightarrow Y$

$I(X;Y)$

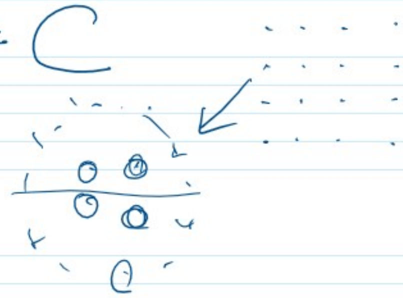
$P_X(x)$?

$x\in\{0,1,2\}$

$P_X(x)$ $\begin{pmatrix}p_0\\p_1\\p_2\end{pmatrix}$ $0\le p_i\le 1$ $\sum p_i=1$

$$\boxed{\max_{P_X(x)} I(X;Y)} = C$$

16-APSK



☐ $I(X;Y) = H(X) - H(X|Y)$

$$= E[-\log_2 P_X(X)] - E[-\log_2 P_{X|Y}(X,Y)]$$

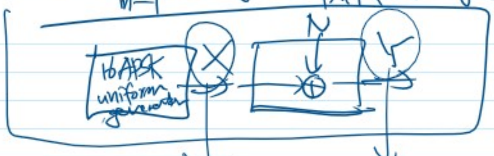
$$= E\left[-\log_2 \frac{P_X(X)}{P_{X|Y}(X,Y)}\right] = E\left[-\log_2 \frac{P_Y(Y)}{P_{Y|X}(Y,X)}\right] = E\left[-\log_2 \frac{f_Y(Y)}{f_{Y|X}(Y,X)}\right]$$

$$= E\left[-\log_2 \frac{P_X(X)\,P_Y(Y)}{\ldots}\right] = I(Y;X)$$

$f_{Y|X}(y|x)$

$$= E\left[-\log_2 \frac{P_X(X)\,P_Y(Y)}{P_{X,Y}(X,Y)}\right] = I(Y;X)$$

$$\simeq \sum_{m=1}^{10^6}\left\{-\log_2 \frac{P_X(x[m])\,P_Y(y[m])}{P_{X,Y}(x[m],y[m])}\right\}\frac{1}{10^6}$$



$X[m]=x[m] \quad Y[m]=y[m]$

$$\to f_{Y|X}(y|x) = \frac{1}{\pi\sigma^2}e^{-\frac{|y-x|^2}{\sigma^2}}$$

$$f_Y(y) = E\left[f_{Y|X}(y|X)\right]$$

$$= \sum_{x' \in \mathcal{X}^{16}} \frac{1}{16}\boxed{f_{Y|X}(y|x')}$$

$$= \sum_{x' \in \mathcal{X}^{16}} \frac{1}{16}\cdot\frac{1}{\pi\sigma^2}e^{-\frac{|y-x'|^2}{\sigma^2}}$$

☐ X con. Y discrete

$$f_X(x)\,P_{Y|X}(y|x) = P_Y(y)\,f_{X|Y}(x|y)$$

☐ X continuous

$$h(X) \triangleq E\left[-\log_2 f_X(X)\right]$$

differential entropy of

$$= \int_{-\infty}^{\infty}\left(-\log_2 f_X(x)\right) f_X(x)\,dx$$

☐ X cont. Y cont

$$h(X,Y) = E\left\{-\log_2 f_{X,Y}(X,Y)\right\}$$

$$h(X|Y) = E\left\{-\log_2 f_{X|Y}(X|Y)\right\}$$

$$= \iint -\log_2 f_{X|Y}(x|y)\,f_{X,Y}(x,y)\,dx\,dy$$

$$I(X;Y) = h(X) - h(X|Y)$$

$$= E\left\{-\log \frac{f_X(X)\,f_Y(Y)}{f_{X,Y}(X,Y)}\right\}$$

☐ 16-APSK

$$I(X;Y) \simeq \frac{1}{10^6}\sum_{m=1}^{10^6}\left\{-\log_2 \frac{f_Y(y[m])}{f_{Y|X}(y[m]|x[m])}\right\} \to \frac{1}{16}\Sigma \cdots$$