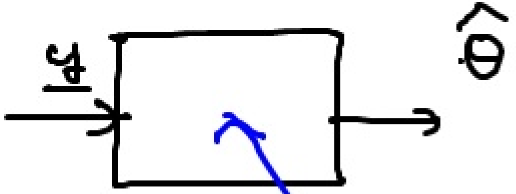
Lec. 03

Announcements

- Quiz #1 on this Friday,
- " Covers upto today's Lecture.
- Problems will look similar to those appeared previous years.

Review

MAP detector



M-ary hypothesis testing

$\min \Pr(\Theta \neq \hat{\Theta})$ ← criterion.

$$\hat{\Theta} = \underset{i \in \{1,2,\cdots,M\}}{\arg\max} \Pr(\Theta = i \mid \underline{Y} = \underline{y})$$

Q. Find the MAP detector given

$$\begin{cases} f_{\underline{Y}|\Theta=1}(\underline{y}) = \dfrac{1}{2\pi\sigma^2}\exp\left(-\dfrac{\|\underline{y}-\underline{s}_1\|^2}{2\sigma^2}\right) = f_{\underline{Y}|\Theta}(\underline{y}|1) \\ f_{\underline{Y}|\Theta=2}(\underline{y}) = \dfrac{1}{2\pi\sigma^2}\exp\left(-\dfrac{\|\underline{y}-\underline{s}_2\|^2}{2\sigma^2}\right) = f_{\underline{Y}|\Theta}(\underline{y}|2) \\ \Pr(\Theta=1)=p \\ \Pr(\Theta=2)=1-p \end{cases}$$

A. Since the decision rule reduces to

$$f_{\underline{Y}|\Theta=1}(\underline{y})\Pr(\Theta=1) \underset{\hat{\Theta}=2}{\overset{\hat{\Theta}=1}{\gtrless}} f_{\underline{Y}|\Theta=2}(\underline{y})\Pr(\Theta=2)$$

p.3

$$\frac{1}{\cancel{2\pi\sigma^2}} \exp\left(-\frac{\|\underline{y}-\underline{s}_1\|^2}{2\sigma^2}\right) p \underset{\hat\theta=2}{\overset{\hat\theta=1}{\gtrless}} \frac{1}{\cancel{2\pi\sigma^2}} \exp\left(-\frac{\|\underline{y}-\underline{s}_2\|^2}{2\sigma^2}\right)(1-p)$$

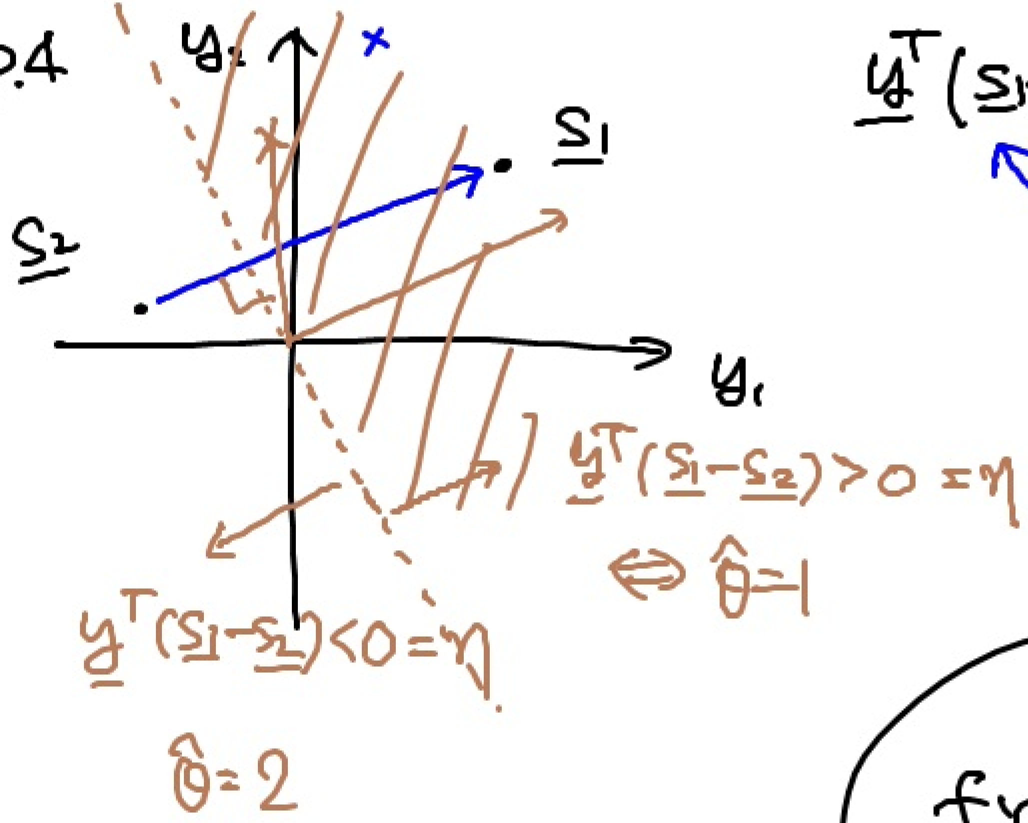
$$\frac{\exp\left(-\frac{\|\underline{y}-\underline{s}_1\|^2}{2\sigma^2}\right)}{\exp\left(-\frac{\|\underline{y}-\underline{s}_2\|^2}{2\sigma^2}\right)} \underset{\hat\theta=2}{\overset{\hat\theta=1}{\gtrless}} \frac{1-p}{p}$$

$$-\frac{\|\underline{y}-\underline{s}_1\|^2}{2\sigma^2} + \frac{\|\underline{y}-\underline{s}_2\|^2}{2\sigma^2} \gtrless \ln\frac{1-p}{p}$$

$$-\left(\cancel{\|\underline{y}\|^2} - 2\underline{y}^T\underline{s}_1 + \|\underline{s}_1\|^2\right) + \left(\cancel{\|\underline{y}\|^2} - 2\underline{y}^T\underline{s}_2 + \|\underline{s}_2\|^2\right) \gtrless 2\sigma^2 \ln\frac{1-p}{p}$$

$$\boxed{\underline{y}^T(\underline{s}_1-\underline{s}_2) \gtrless \overbrace{\sigma^2 \ln\frac{1-p}{p} + \frac{\|\underline{s}_1\|^2 - \|\underline{s}_2\|^2}{2}}^{\eta}}$$

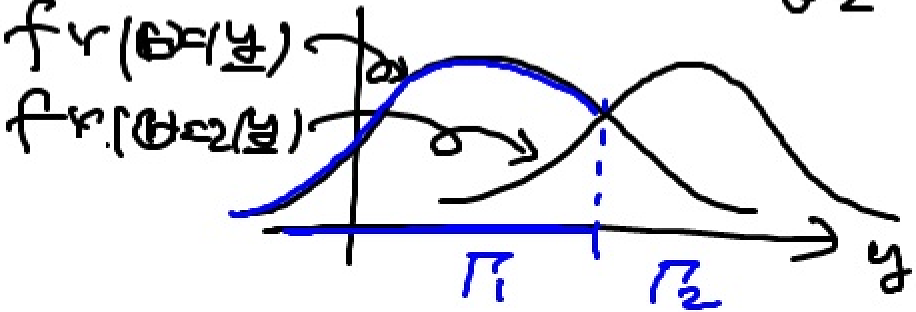
P.4



$$\underline{y}^T(\underline{s}_1-\underline{s}_2) \underset{\hat{\theta}=2}{\overset{\hat{\theta}=1}{\gtrless}} \eta$$

Binary hypothesis testing

$$\frac{f_{Y|\Theta=1}(\underline{y})\,Pr(\Theta=1)}{f_{Y|\Theta=2}(\underline{y})\,Pr(\Theta=2)} \underset{\hat{\theta}=2}{\overset{\hat{\theta}=1}{\gtrless}} 1$$



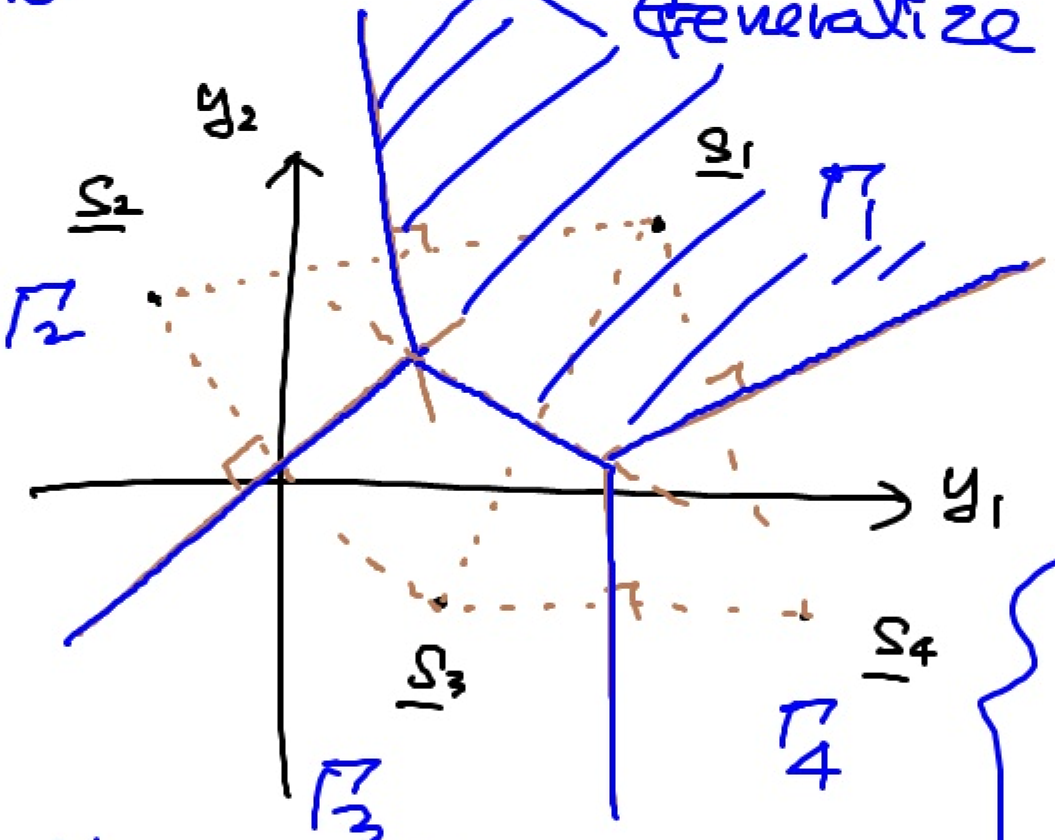
~~p.5~~ p.6 Preview

Specialize

Generalize

MAP

General Bayesian hypothesis testing



Voronoi Regions

$\underline{Y} = \underline{S}_i + \underline{N}$, $\underline{N} \sim N(\underline{0}, \sigma^2 I)$

$\hat{\theta} = \arg\min_i \|\underline{y} - \underline{S}_i\|$

White Gaussian.

ML

= minimum Euclidean distance decision rule

p.7

$$\Pr(\Theta \neq \hat{\Theta}) = \sum_{i=1}^{M} \Pr(\Theta \neq \hat{\Theta} \mid \Theta = i)\Pr(\Theta = i)$$

$$= \sum_{i=1}^{M} \left(\Pr(\hat{\Theta} \neq i \mid \Theta = i)\right)\Pr(\Theta = i)$$

$$= \sum_{i=1}^{M} \left(\sum_{j=1}^{M} (1-\delta_{i,j}) \Pr(\hat{\Theta} = j \mid \Theta = i)\right)\Pr(\Theta = i)$$

Interpret it as cost of "say $\hat{\theta} = j$ when $\theta = i$".

MAP criterion

= uniform cost

P8

$\theta \backslash \hat{\theta}$	P	NP
P	Not an error	type A
NP	typeB	Not an error

$\theta \backslash \hat{\theta}$	1	2	$\cdots$	M
1	X			
2		X		
$\vdots$			$\ddots$	
M				X

Binary $\Rightarrow$ 2 types of errors

Q. M-ary $\Rightarrow$ (M^2-M) types of errors.

p.9

$$\sum_{i=1}^{M} \sum_{j=1}^{M} C_{j,i} \Pr(\hat{\Theta}=j \mid \Theta=i) \Pr(\Theta=i)$$

$$= \sum_{i=1}^{M} \sum_{j=1}^{M} C_{j,i} \Pr(\Theta=i, \hat{\Theta}=j)$$

$$= E\left[C_{\hat{\Theta},\Theta}\right]$$

기대값 $= \sum$ 값 $\times$ 확률

$$E[g(X,Y)]$$

$$= \sum_x \sum_y g(x,y) \Pr(X=x, Y=y)$$