

Lec. 2h

Announcements



; No quiz tomorrow.



; No class meeting

tomorrow
Sept. 27.



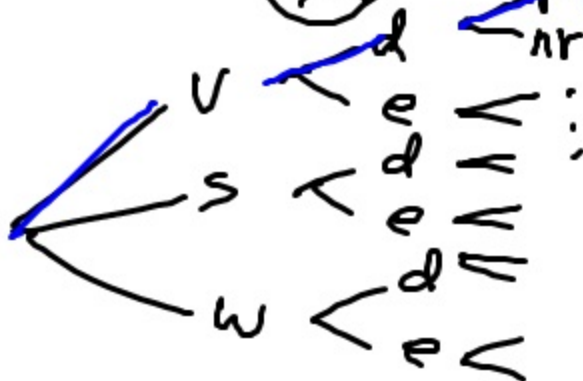
; Make-up Friday Sept. 28

MAP.

General Bayesian Detection

$E[\hat{C}_{\theta, \theta}]$

Review



Preview



COST $\hat{C}_{\theta, \theta}$

$1 - \delta_{\hat{\theta}, \theta}$ (assigned
uniform cost

p.2

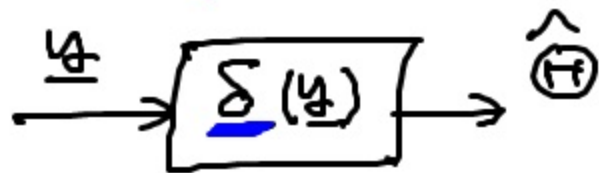
$$\min_{\underline{\delta}(\underline{y})} E[C(\hat{\Theta}, \Theta)]$$

$\equiv$ minimize the
expected cost
average(d)

$\equiv$ minimize the
Bayes
risk

$\equiv$ find the Bayesian detector.

randomized decision rule for
general M-ary detector



$$\underline{\delta}(\underline{y}) \triangleq \begin{bmatrix} P_r(\hat{\Theta}=1 | Y=\underline{y}) \\ P_r(\hat{\Theta}=2 | Y=\underline{y}) \\ \vdots \\ P_r(\hat{\Theta}=M | Y=\underline{y}) \end{bmatrix}$$

p.3

$$\hat{\Theta}_{\text{MAP}} = \underset{i \in \{1, 2, \dots, M\}}{\text{argmax}} \Pr(\Theta = i | Y = y)$$

$$C_{\hat{\Theta}, \Theta} = 1 - \delta_{\hat{\Theta}, \Theta} = f_{Y/\Theta=i}(y) \Pr(\Theta=i)$$

$$\hat{\Theta} = \underset{i \in \{1, 2, \dots, M\}}{\text{argmin}} E[C_{i, \Theta} | Y = y]$$

/ex/ MAP

$$\begin{aligned} & \underset{i \in \{1, 2, \dots, M\}}{\text{argmin}} E[1 - \delta_{i, \Theta} | Y = y] \\ &= \underset{i \in \{1, 2, \dots, M\}}{\text{argmax}} E[\delta_{i, \Theta} | Y = y] \\ &= \underset{i \in \{1, 2, \dots, M\}}{\text{argmax}} \sum_{\theta=1}^M \delta_{i, \theta} \Pr(\Theta = \theta | Y = y) \end{aligned}$$

P4

$$E[C(\hat{H}, H)] = \sum_{\theta=1}^M E[C(\hat{H}, \theta) | H = \theta] P(H = \theta)$$

Total Expectation Theorem

conditional risk.

$R(\delta/\theta)$

MAP

$$P(\hat{H} \neq H) = \sum_{\theta=1}^M P(\hat{H} \neq \theta | H = \theta) P(H = \theta)$$

tradeoff