Lec. 06

Announcement

Quiz on this Friday.

point, interval > estimation

Review

- vector
  - d
    - r
    - n.r
  - e
    - r
    - n.r
- sequence
  - d
  - e
- waveform
  - d
  - e

r — min P_e $\Rightarrow$ MAP

r — min $E[C_{\hat{\Theta},\Theta}] \Rightarrow$

$$\hat{\theta} = \arg\max_i \Pr(\Theta = i \mid \underline{Y} = \underline{y})$$

$$\hat{\theta} = \arg\min_i E[C_{i,\Theta} \mid \underline{Y} = \underline{y}]$$

$C_{i,0} \Pr(\Theta = 0 \mid Y = y)$

$+ C_{i,1} \Pr(\Theta = 1 \mid Y = y)$

$\underline{Y}$ $\quad$ $\underline{\theta} \in \Omega$ $\quad$ $\underline{\Theta}$, $\underline{\Theta}$

- Binary Bayesian Hypothesis Test.

  - observation $H_0 : \underline{Y} \sim f_{\underline{Y}|\Theta=0}(\underline{y})$

    vs.

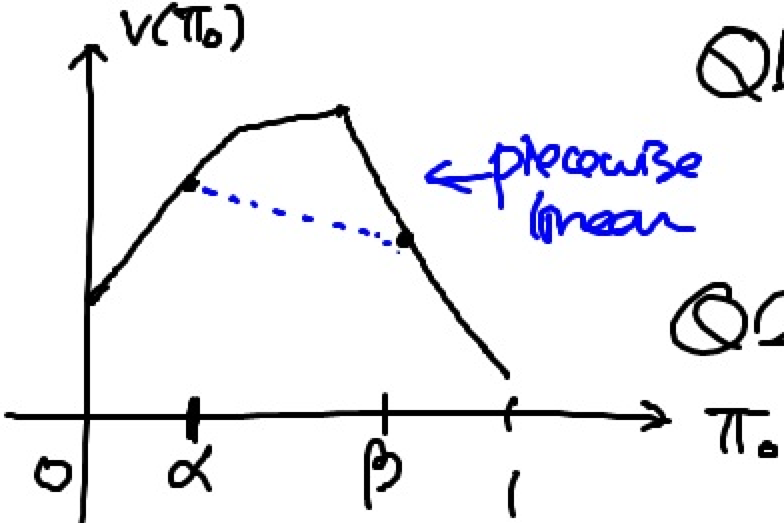
    $H_1 : \underline{Y} \sim f_{\underline{Y}|\Theta=1}(\underline{y})$

  - $\pi_0 \triangleq \Pr(\Theta=0)$, $\pi_1 \triangleq \Pr(\Theta=1) = 1-\pi_0$

  - $C_{\hat{\theta},\theta}$ $\quad \theta, \hat{\theta} \in \{0, 1\}$.

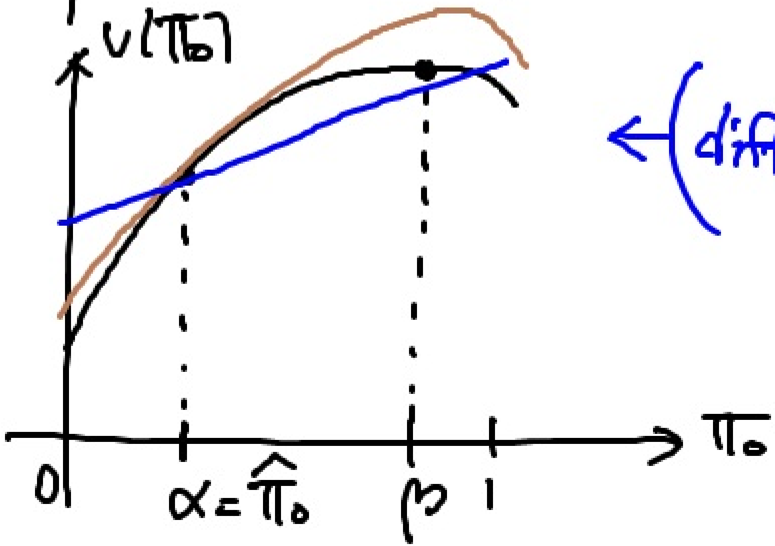
  - $\delta(\underline{y})$

- $\delta_{\pi_0} \triangleq \underset{\delta}{\arg\min}\, r(\delta;\pi_0)$, $\quad V(\pi_0) \triangleq r(\delta_{\pi_0}, \pi_0)$



Q1. $V(\pi_0)$ is a convex, concave function of π_0

Q2. If $V(\pi_0)$ is concave, then $V(\pi_0)$ is continuous.



← (differentiable)

Q3. $V(\pi_0) \triangleq r(\delta_{\pi_0}, \pi_0)$ vs. $r(\delta_{\hat{\pi}_0}, \pi_0)$ as fts of π_0.

p.4 Property of $r(\delta_{\hat{\pi}_0}, \pi_0)$ as a function of π_0

$$r(\delta_{\hat{\pi}_0}, \pi_0) \triangleq \underset{\delta_{\hat{\pi}_0}, \pi_0}{E}\left[C_{\hat{\Theta}, \Theta}\right] = E\left\{E\left[C_{\hat{\Theta}, \Theta} \mid \Theta\right]\right\}$$

π_0: $Pr(\Theta = 0)$

$$= \underset{\delta_{\hat{\pi}_0}}{E}\left[C_{\hat{\Theta}, \Theta} \mid \Theta = 0\right] Pr(\Theta = 0)$$

$E[X] = E\{E[X \mid Y]\}$

$$+ \underset{\delta_{\hat{\pi}_0}}{E}\left[C_{\hat{\Theta}, \Theta} \mid \Theta = 1\right] Pr(\Theta = 1)$$

$$\triangleq r(\delta_{\hat{\pi}_0} \mid \Theta = 0)\, \pi_0 + r(\delta_{\hat{\pi}_0} \mid \Theta = 1) \cdot (1 - \pi_0)$$