

Lec. #7

Announcements

Good news: No class tomorrow

Bad news: Quiz #1 only.

Review Binary Bayesian Hypothesis Test

$$H_0: Y \sim f_Y(\Theta=0|Y)$$

vs

$$H_1: Y \sim f_Y(\Theta=1|Y)$$

$$\pi_0 \triangleq \Pr(\Theta=0), \quad \pi_1 \triangleq \Pr(\Theta=1) = 1 - \pi_0$$

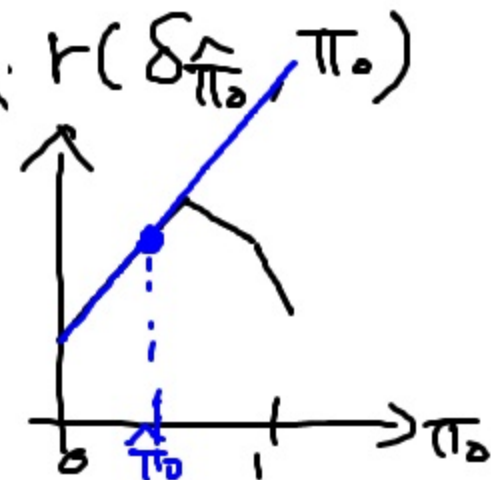
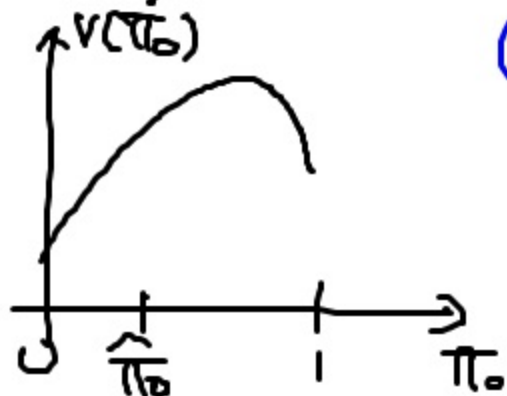
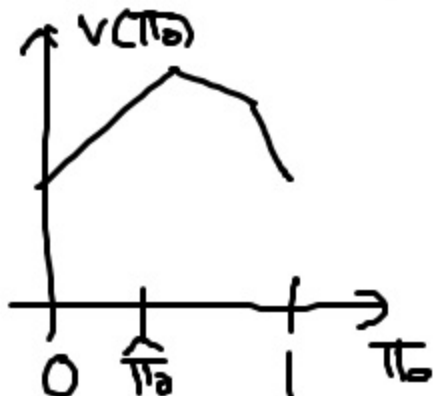
$$C_{\hat{\Theta}, \Theta}$$

$$\delta_{\pi_0} \triangleq \argmin_{\delta} E[C_{\hat{\Theta}, \Theta}]$$

p.2

$$V(\pi_0) \triangleq r(\delta_{\pi_0}, \pi_0)$$

$V(\pi_0)$ is concave
($\Rightarrow$ continuous)



$$\begin{aligned} r(\delta_{\hat{\pi}_0}, \pi_0) &= E[C_{\theta, \theta}]_{\delta_{\hat{\pi}_0}, \pi_0} \\ &= E\{E[C_{\theta, \theta} | \theta]\}_{\delta_{\hat{\pi}_0}, \pi_0} \\ &= E_{\theta \sim \pi_0}[C_{\theta, \theta}(\theta=0)]\pi_0 + E_{\theta \sim (1-\pi_0)}[C_{\theta, \theta}(\theta=1)](1-\pi_0) \\ &\quad \pi_0 = P(\theta=0) \end{aligned}$$

p.3

$$\min_x \left[\min_y \begin{array}{l} f(x, y) \\ \text{s.t. } y \in \Omega_x \end{array} \right]$$

← Double minimization problem.

≡ Nested "

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$g(x)$

inner optimization

outer optimization

$$\min_{\delta} \max_{\theta} \left\{ E_{\delta} [C(\hat{\theta}, \theta) | \theta=0], E_{\delta} [C(\hat{\theta}, \theta) | \theta=1] \right\}$$

$$\min_{\delta} \max \left\{ R(\delta; \theta=0), R(\delta; \theta=1) \right\}$$

p.4

$$\min_{x \in [a,1]} \max_{y \in [a,1]} f(x,y)$$

$$\geq \max_{y \in [a,1]} \min_{x \in [a,1]} f(x,y)$$

