Lec. 08

Announcements

1. No lecture on this Thursday.

   But we'll have

   Quiz #2

   recap session : review of previous exam problems

   instead.

2. Two consecutive Make-up classes on this Friday.

3. For other updates, see Schedule/Downloads on the course webpage.

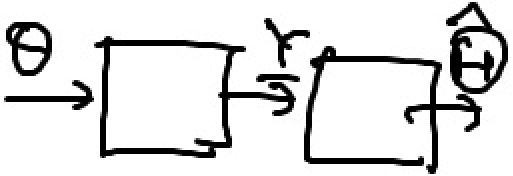
p2. Review

$$\delta_{\underline{\min}\underline{\max}} = \arg\min_{\delta} \max\{\, r(\delta;\theta=0),\ r(\delta;\theta=1)\,\}$$

where conditional risks are given by

$$r(\delta;\theta=0) = C_{0,0}\Pr(\hat{\Theta}=0;\theta=0) + C_{1,0}\Pr(\hat{\Theta}=1;\theta=0)$$

$$r(\delta;\theta=1) = C_{0,1}\Pr(\hat{\Theta}=0;\theta=1) + C_{1,1}\Pr(\hat{\Theta}=1;\theta=1)$$



and $\Pr(\hat{\Theta}=j;\theta=i) = \int_{\underline{y}} \Pr(\hat{\Theta}=j \mid \underline{Y}=\underline{y}; \cancel{\theta=i})\, f_{\underline{Y};\theta=i}(\underline{y})\, d\underline{y}$

the prob. is given by

$$= \int_{\underline{y}} \begin{cases} \delta(\underline{y}) & \text{if } j=1 \\ 1-\delta(\underline{y}) & \text{if } j=0 \end{cases} f_{\underline{Y};\theta=i}(\underline{y})\, d\underline{y}$$

p.3

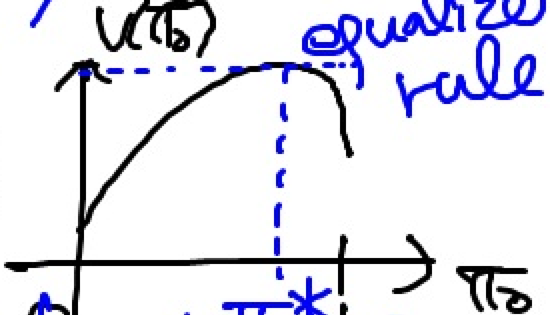
$$\min_{\delta} \left(\max \{ r(\delta;\theta=0),\ r(\delta;\theta=1) \} \right)$$

$$= \min_{\delta} \max_{0\le\pi_0\le 1} r(\delta,\pi_0) \qquad \text{where } \pi_0 \triangleq \Pr(\Theta=0)$$

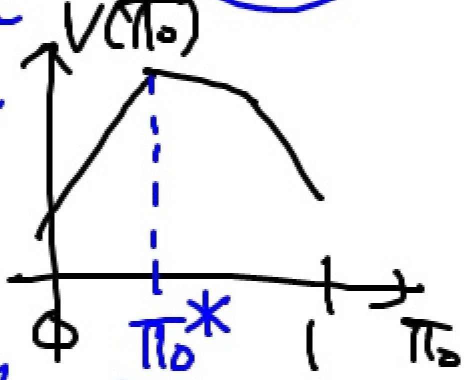
$$\ge \max_{0\le \pi_0\le 1} \underbrace{\left(\min_{\delta} r(\delta,\pi_0)\right)}_{=V(\pi_0)}$$

inner optimization ...

$$= \max_{0\le\pi_0\le 1} V(\pi_0) = V(\pi_0^*)$$

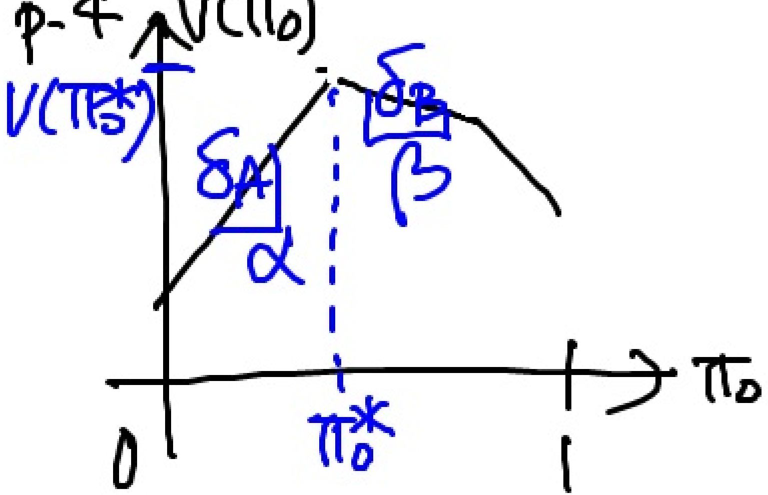


equalizer rule.



where π_0^* is called the least favorable prior.

p-4



$$\delta_A(\underline{y}) \triangleq \Pr(\hat{H}=1 \mid \underline{Y}=\underline{y}\,;\delta_A)$$

$$\delta_B(\underline{y}) \triangleq \Pr(\hat{H}=1 \mid \underline{Y}=\underline{y}\,;\delta_B)$$

$$\Pr(\delta=\delta_A)=\gamma$$

$$\Pr(\delta=\delta_B)=1-\gamma$$

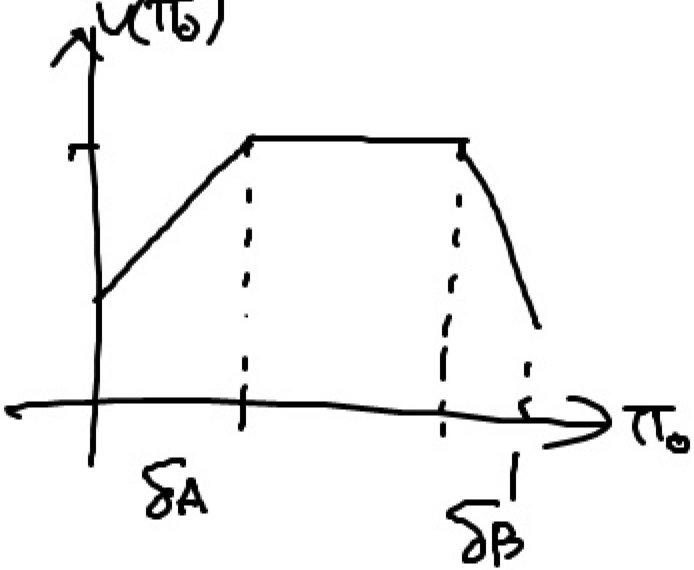
$$\delta(\underline{y}) = \Pr(\hat{H}=1 \mid \underline{Y}=\underline{y}\,;\delta)$$

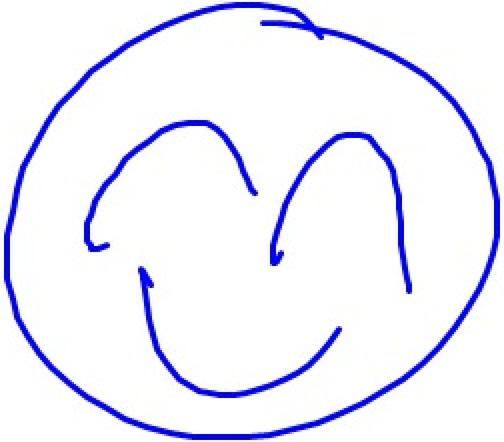
$$= \Pr(\hat{H}=1 \mid \underline{Y}=\underline{y},\ \delta=\delta_A)\Pr(\delta=\delta_A \cancel{\mid \underline{Y}=\underline{y}})$$

$$+\Pr(\hat{H}=1 \mid \underline{Y}=\underline{y},\ \delta=\delta_A)\Pr(\delta=\delta_B \cancel{\mid \underline{Y}=\underline{y}})$$

$$= \gamma\,\delta_A(\underline{y}) + (1-\gamma)\,\delta_B(\underline{y}), \qquad \gamma^*, \delta_A, \delta_B$$

R5





minimax criterion.

inner optimization (discrete $\rightarrow$ continuous)

minimax $\geq$ maxmin

least favorable prior, equalizer rule. randomized rule p.6

$$\min_{x\in\mathbb{R}} \quad x^2+2x-6 \triangleq f(x)$$

Somehow, $f(x) \geq -10, \ \forall x$

a loose lower bound.

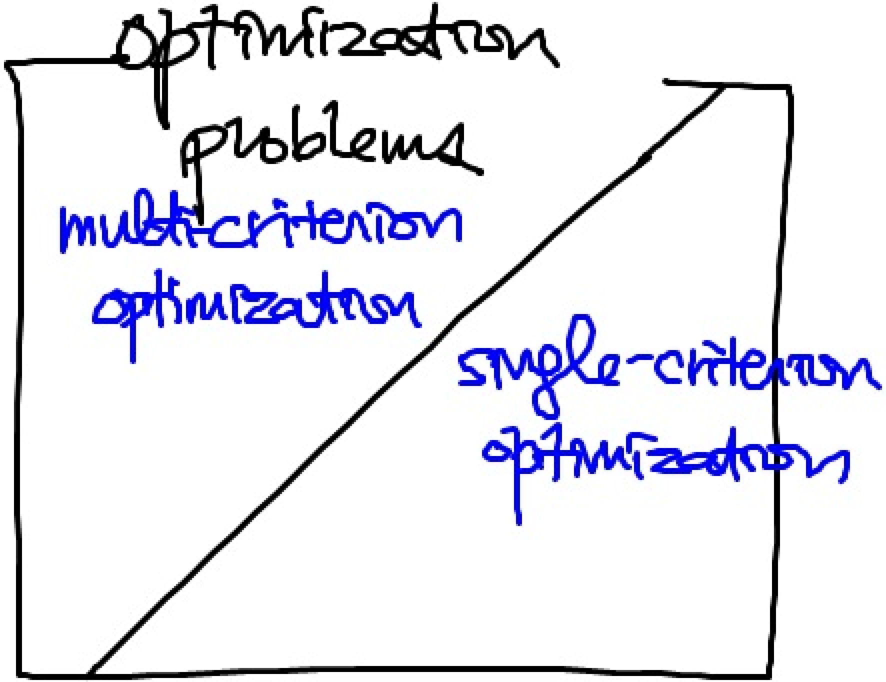
$\updownarrow$

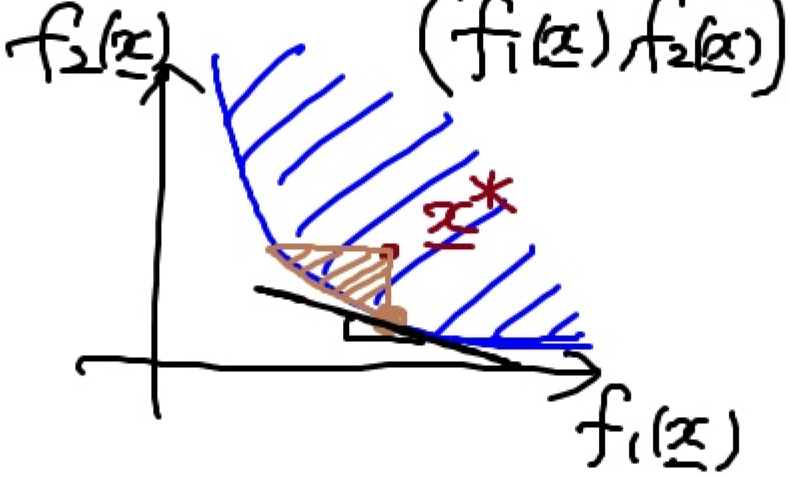
tight

$f(x) \geq -7, \ \forall x$

$f(-1) = -7.$

p.1)





$$\begin{cases} \min\limits_{\underline{x}} f_1(\underline{x}) \\ \min\limits_{\underline{x}} f_2(\underline{x}) \end{cases}$$

$$\underline{1}^T \underline{x} = 10억\$$$

$$\underline{x} = \begin{bmatrix} \text{가상자산} \\ \text{SK전} \\ \text{secuvt} \\ \text{은수} \\ \vdots \end{bmatrix}$$