Lec. 09

Announcement

Midterm Exam #1 on Oct. 4.

It will cover upto the N-P detector.

Review

The Neyman-Pearson optimality criterion

- Binary
- non-random parameter

No $Pr(\theta=0)$, $Pr(\theta=1)$.

$\theta \backslash \hat{\theta}$	0	1
0		F(A)
1	M	D

P_{FA}, P_F

P_D, P_M

p.2 the N-P opt. criterion $\max P_D$ st. $P_F \le \alpha$ $\equiv$ $\min P_M$ st. $P_F \le \alpha$

- ~~3~~ 4 steps to tackle an optimization problem.

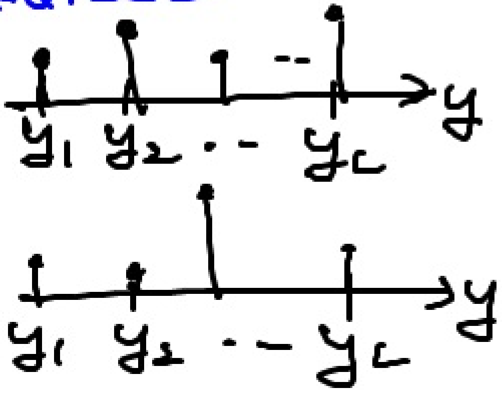
1. Formulation
   - criterion ← $\min f(\underline{x})$
   - parameter ← $\underline{x}$
   - constraint ← s.t. $\underline{x} \in \Omega$.
2. Check existence & uniqueness
3. Find {a / the / all} solution(s).
4. Evaluate the performance of {a / the / all} solution(s).

p.3 Simplify! Visualize!

Y a random variable $\leftarrow$ discrete / continuous / mixed.

Y a discrete r.v.

p_0 p_1



$H_0 : Y \sim p_0(y)$ $\quad y \in \{y_1, y_2, \dots, y_L\}$

vs

$H_1 : Y \sim p_1(y)$

$\mathcal{Y}$ finite.

$$\max_{\underline{\delta}} \; P_D \quad \text{s.t.} \quad P_F \le \alpha$$

$$\underline{\delta} = \begin{bmatrix} \delta(y_1) \\ \delta(y_2) \\ \vdots \\ \delta(y_L) \end{bmatrix}, \quad 0 \le \delta(y_\ell) \le 1$$

p4 Q. Rewrite P_D & P_F in terms $\underline{p}_0, \underline{p}_1$, and $\underline{\delta}$.

A. $$P_D = \Pr(\hat{\Theta}=1 \,;\, \theta=1) = \sum_{l=1}^{L} \Pr(\hat{\Theta}=1 \mid Y=y_l \,\cancel{;\, \theta=1}) \times \Pr(Y=y_l \,;\, \theta=1)$$
$$= \underline{\delta}^T \underline{p}_1$$

$$P_F = \Pr(\hat{\Theta}=1 \,;\, \theta=0) = \sum_{l=1}^{L} \Pr(\hat{\Theta}=1 \mid Y=y_l \,\cancel{;\, \theta=0}) \times \Pr(Y=y_l \,;\, \theta=0)$$
$$= \underline{\delta}^T \underline{p}_0$$

Korean

高手

↕

下手

Chinese

新手

Japanese

上手

p.5

$$\max_{\underline{\delta}} \ \underline{\delta}^T \underline{p}_1 \iff \sum_{l=1}^{L} \delta_l p_{1,l} = \sum_{l=1}^{L} \tilde{\delta}_l \left(\frac{p_{1,l}}{p_{0,l}}\right)$$

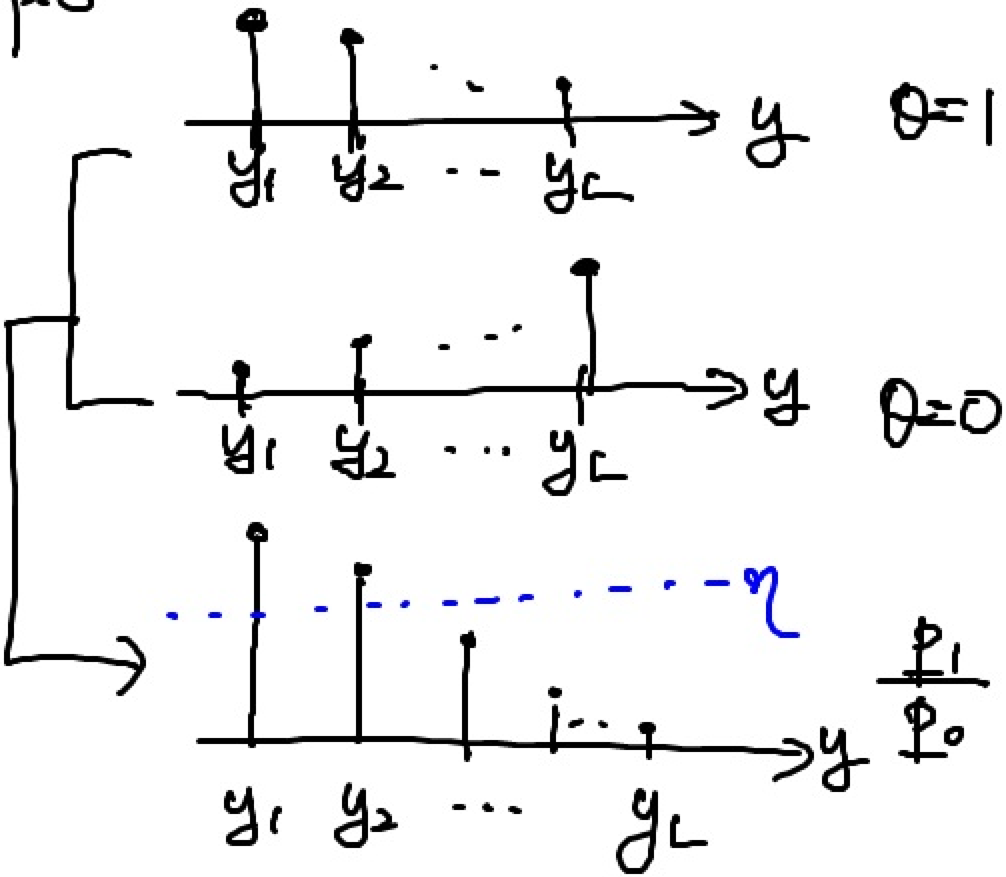
$$\text{s.t.} \ \underline{\delta}^T \underline{p}_0 \le \alpha \iff \sum_{l=1}^{L} \underbrace{\delta_l p_{0,l}}_{\triangleq \tilde{\delta}_l} \le \alpha \iff \underline{\tilde{\delta}}^T \underline{1} \le \alpha$$

commonalities
differences

$$0 \le \delta_l \le 1 \iff 0 \le \tilde{\delta}_l \le p_{0,l}, \ \forall l$$

$$\boxed{\begin{aligned} &\max_{\underline{\tilde{\delta}}} \ \sum_{l=1}^{L} \tilde{\delta}_l \left(\frac{p_{1,l}}{p_{0,l}}\right) \\ &\text{s.t.} \ \sum_{l=1}^{L} \tilde{\delta}_l \le \alpha, \quad 0 \le \tilde{\delta}_l \le p_{0,l}, \end{aligned}}$$

p6



$$\frac{p_1}{p_0} = p_1 ./ p_0$$