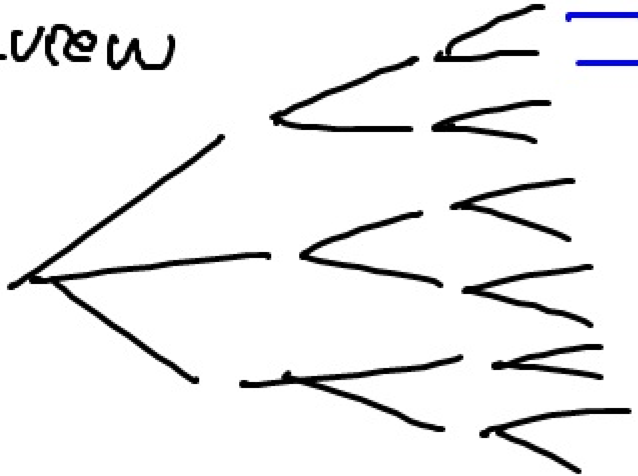
Lec. 11

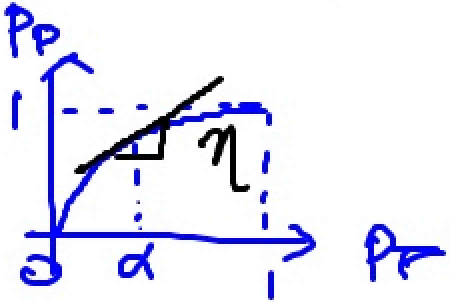
Announcement

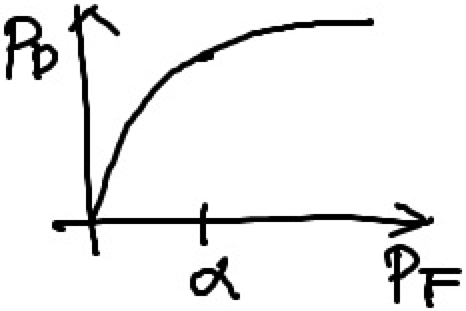
- Make-up class meetings for Lecs #12 & #13 on this Friday.

Review



ROC





persistently

exhaustive search (simplify)

$$P_D = \int_{\underline{y} \in \left\{ \underline{y} : \frac{f_{\underline{Y};H_1}(\underline{y})}{f_{\underline{Y};H_0}(\underline{y})} \geq \eta \right\}} f_{\underline{Y};H_1}(\underline{y})\, d\underline{y}$$

$$= \int_{\underline{y}} 1_{\left\{ \underline{y} : \text{———} \geq \eta \right\}}(\underline{y})\, f_{\underline{Y};H_1}(\underline{y})\, d\underline{y}$$

$$= E_{H_1}\left[1_{\{ \quad \}}(\underline{Y}) \right]$$

$$P_F = E_{H_0}\left[1_{\{ \quad \}}(\underline{Y}) \right]$$

Too good to be true.

p.3

$$P_D = E_{H_1}\left[1_{\{\underline{y}:\ \_\_\_\_ \ge \eta\}}(\underline{Y})\right]$$

$$= \int_{\underline{y}} 1_{\{\underline{y}:\ \_\_\_\_ \ge \eta\}}(\underline{y}) \frac{f_{\underline{Y};H_1}(\underline{y})}{f_{\underline{Y};H_0}(\underline{y})} f_{\underline{Y};H_0}(\underline{y})\, dy$$

$$= E_{H_0}\left[1_{\{\underline{y}:\ \_\_\_\_ \ge \eta\}}(\underline{Y}) \frac{f_{\underline{Y};H_1}(\underline{Y})}{f_{\underline{Y};H_0}(\underline{Y})}\right]$$

$$\boxed{S} \xrightarrow{\underline{Y}} \boxed{\Lambda(\underline{y})} \to \Lambda \qquad \Lambda(\underline{y}) \triangleq \frac{f_{\underline{Y};H_1}(\underline{Y})}{f_{\underline{Y};H_0}(\underline{Y})}$$

$$= E_{H_0}\left[1_{\{\underline{y}:\ \Lambda(\underline{y}) \ge \eta\}}(\underline{Y}) \cdot \Lambda(\underline{Y})\right]$$

$$= E_{H_0}\left[1_{\{\lambda:\ \lambda \ge \eta\}}(\Lambda) \cdot \Lambda\right] = \boxed{\int_\eta^\infty \lambda f_{\Lambda;H_0}(\lambda)\, d\lambda}$$

p.4

$$P_D = \int_\eta^\infty \lambda f_{\Lambda;H_0}(\lambda)\,d\lambda$$

$$P_F = \int_\eta^\infty 1 \cdot f_{\Lambda;H_0}(\lambda)\,d\lambda$$

$$\Rightarrow \frac{dP_D}{d\eta} = -\eta f_{\Lambda;H_0}(\eta)$$ by Leibniz's rule

$$\frac{dP_F}{d\eta} = - f_{\Lambda;H_0}(\eta)$$

$$\Rightarrow \frac{dP_D}{dP_F} = \eta.$$

p5. the Composite hypothesis testing problem

$$H_0: \underline{Y} = \underline{N}$$

vs

$$H_1: \underline{Y} = \Theta \underline{1} + \underline{N} \quad \text{w/} \quad f_\Theta(\theta)$$

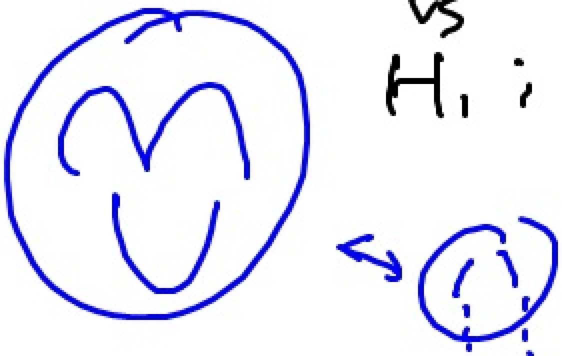
the Nuisance parameter

can be converted to the simple hypothesis testing problem

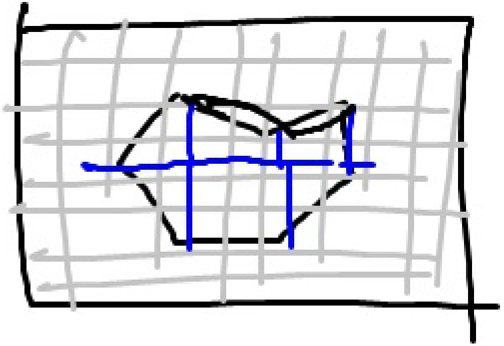
$$H_0: \underline{Y} \sim f_{\underline{N}}(\underline{y})$$

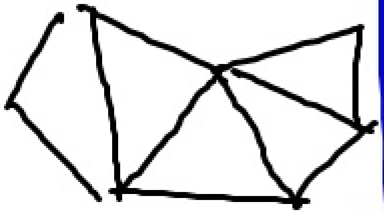
vs

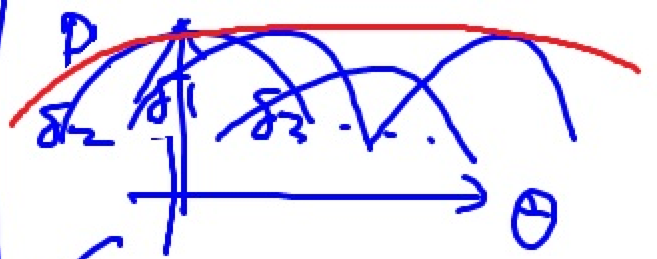
$$H_1: \underline{Y} \sim \int_{-\infty}^{\infty} f_{\underline{N}}(\underline{y} - \theta \cdot \underline{1})\, f_\Theta(\theta)\, d\theta$$



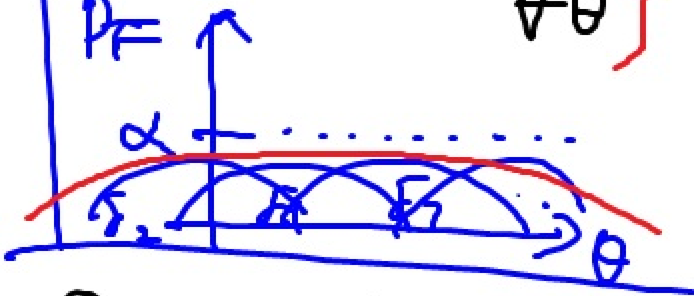
P6







$$\max_{\delta} P_D$$
$$\text{s.t. } \{\delta : P_F \le \alpha, \ \forall \theta\}$$

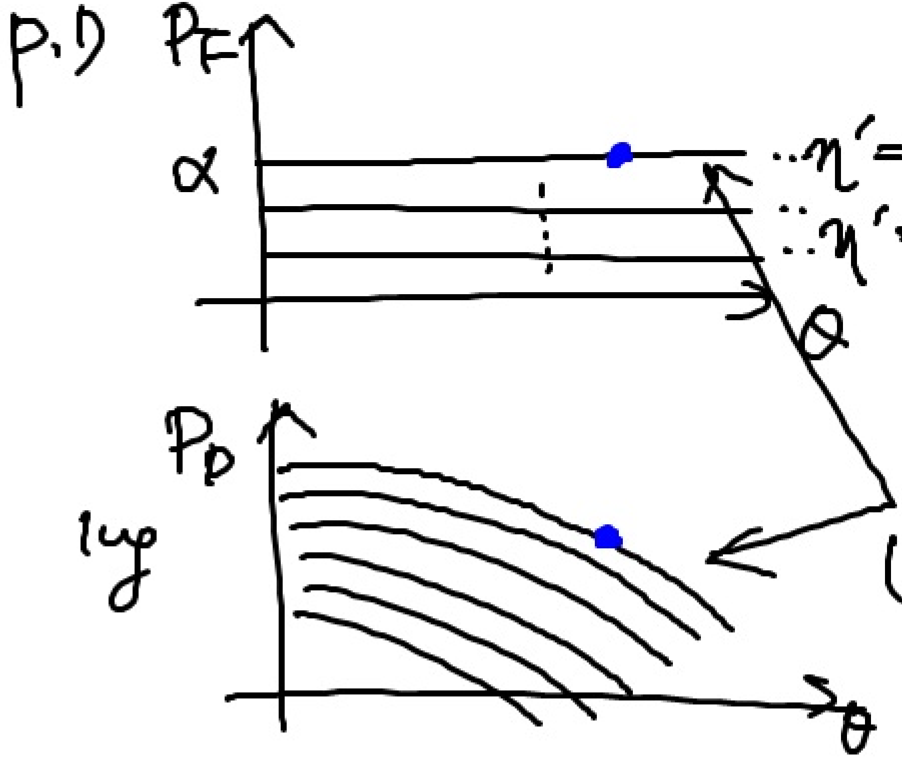


4 steps

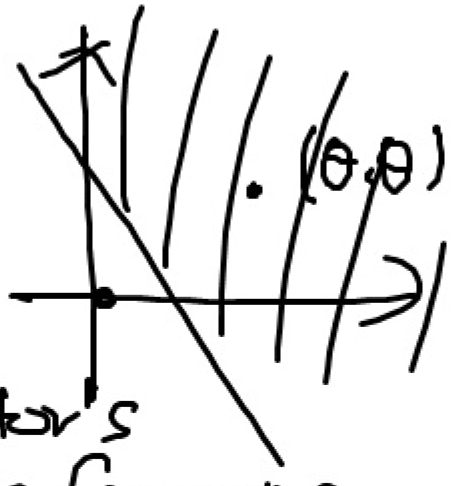
1. Optimality criterion
2. Problem formulation
3. Find a/the solution(s).
4. Evaluate the performance of the solution(s).

If a solution exists, then we call it ~~the~~ UMP detector.

$\{\delta : P_F \le \alpha, \forall \theta\}$



UMP



detector's performance curves