Lec. 12

Announcement:

Midterm exam #1 is graded.

will be given to you after Lec. 12

Review

composite hypothesis testing → abstract

↕

(simple " ")

concrete

ex/

$H_0 : Y \sim N(0, \sigma^2)$

vs

$H_1 : Y \sim N(\theta, \sigma^2)$

$\theta > 0.$

p. 2

the nuisance parameter

$$\max_{\delta(\underline{y})} \quad P_D, \ \forall\theta$$

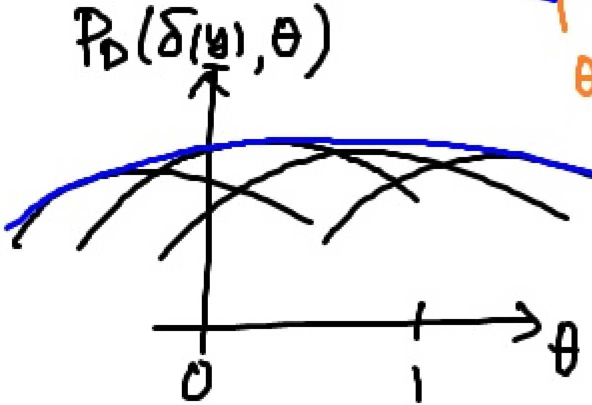
$$\text{s.t.} \quad P_F \le \alpha, \ \forall\theta$$

$$P_F = Pr(\text{say } H_1 ; H_0, \theta)$$

$$= \int_{-\infty}^{\infty} Pr(\text{say } H_1 \mid \underline{Y} = \underline{y} ; \cancel{H_0,} \theta) \, f_{\underline{Y};H_0,\theta}(\underline{y}) \, d\underline{y}$$

$$\equiv \left\{ \delta(\underline{y}) : \cancel{\int_{\Gamma_1} f_{\underline{Y};H_0}(\underline{y}) \, d\underline{y}} \right.$$

$$\equiv \bigcap_{\theta} \left\{ \delta(\underline{y}) : \int_{-\infty}^{\infty} \delta(\underline{y}) f_{\underline{Y};H_0,\theta}(\underline{y}) \, d\underline{y} \right\}$$



the uniformly most powerful (UMP) detector.

p3

$H_0 : Y \sim N(0, \sigma^2)$
vs
$H_1 : Y \sim N(\theta, \sigma^2)$

$\theta > 0$: UMP exists
$\theta < 0$: " "
$\theta \neq 0$: " doesnot exist.

$\equiv$ $Y \sim N(\theta, \sigma^2)$
$H_0 : \theta = 0$
vs
$H_1 : \theta \neq 0$

repeat ARQ
rephrase HAQ

$\simeq$ $H_0 : \theta = 0$
vs
$H_1 : \theta \simeq 0 \ (\theta \neq 0)$

$$\Omega = \bigcap_{\cancel{\theta \neq 0} \atop {\theta \simeq 0 \atop \theta \neq 0}} \left\{ \delta(\underline{y}) : \int_{-\infty}^{\infty} \delta(\underline{y}) f_{Y; H_0, \theta}(\underline{y}) d\underline{y} \right\}$$

p.4

$H_0 : \theta = \theta_0$

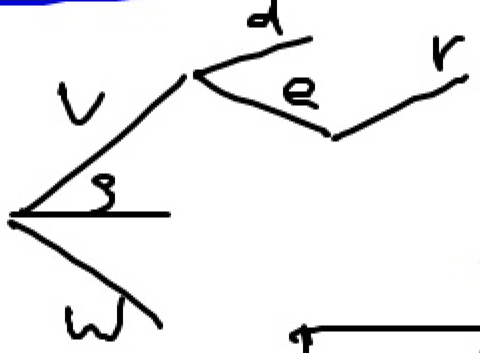
vs

$H_1 : \cancel{\theta \neq \theta_0}, \theta \simeq \theta_0$

$\theta > \theta_0$

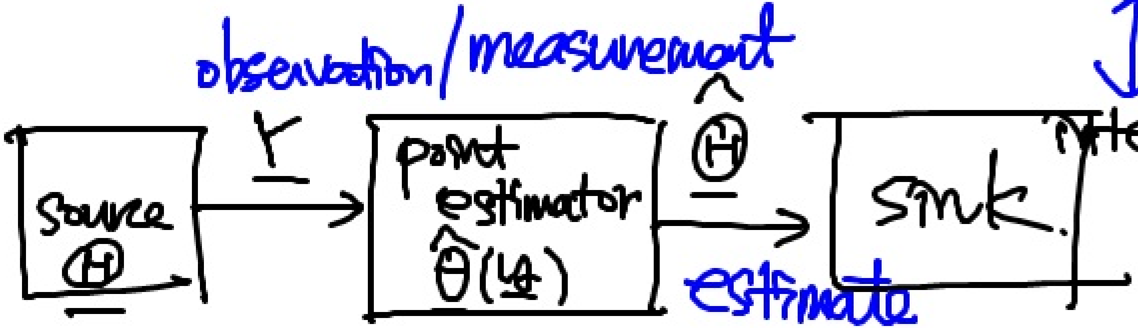
$$P_D(\delta, \theta) = \int_{-\infty}^{\infty} \delta(\underline{y}) f_{\underline{Y};\theta}(\underline{y}) d\underline{y}$$

$$P_D(\delta, \theta_0) = P_F(\delta)$$



: random parameter point estimation

interval



P5.

Signal & System model

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Problem Formulation ← Optimality criterion

Bayes risk minimization

average cost

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Find the/a solution(s)

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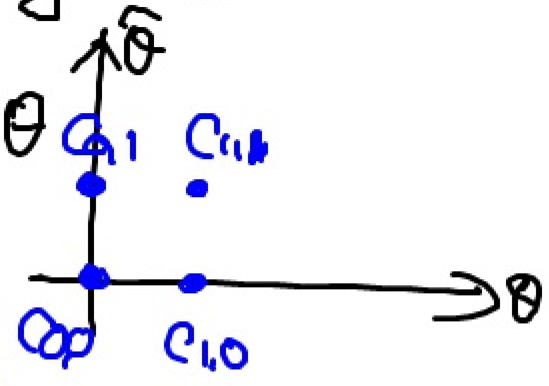
Evaluate the performance

Interpret the solution(s).

$C_{\hat{\theta},\theta}$

$C(\hat{\theta},\theta)$



p6 the Bayes risk $\triangleq E\left[C\left(\hat{\underline{\theta}}(\underline{Y}), \underline{\Theta}\right)\right]$

$$= \int_{\underline{\theta}} \int_{\underline{y}} C\left(\hat{\underline{\theta}}(\underline{y}), \underline{\theta}\right) f_{\underline{Y},\underline{\Theta}}(\underline{y}, \underline{\theta}) \, d\underline{y} \, d\underline{\theta}$$

$$\hat{\underline{\theta}}_{opt}(\underline{y}) = \arg\min_{\hat{\underline{\theta}}(\underline{y})} E\left[C\left(\hat{\underline{\theta}}(\underline{Y}), \underline{\Theta}\right)\right]$$

$f_{\underline{Y}|\underline{\Theta}}(\underline{y}|\underline{\theta})$

$f_{\underline{\Theta}}(\underline{\theta})$

$$= \arg\min_{\hat{\underline{\theta}}(\underline{y})} E\left\{E\left[C\left(\hat{\underline{\theta}}(\underline{Y}), \underline{\Theta}\right) \middle| \underline{Y}\right]\right\}$$

$$f_{\underline{Y},\underline{\Theta}}(\underline{y}, \underline{\theta}) = f_{\underline{Y}|\underline{\Theta}}(\underline{y}|\underline{\theta}) f_{\underline{\Theta}}(\underline{\theta})$$

$$= \arg\min_{\hat{\underline{\theta}}(\underline{y})} E\left[C\left(\hat{\underline{\theta}}(\underline{y}), \underline{\Theta}\right) \middle| \underline{Y} = \underline{y}\right]$$

$f_{\underline{\Theta}|\underline{Y}}(\underline{\theta}|\underline{y})$

$f_{\underline{Y}}(\underline{y})$