Lec. 13

Review

Minuk Kim's Shin Ramen Problem.

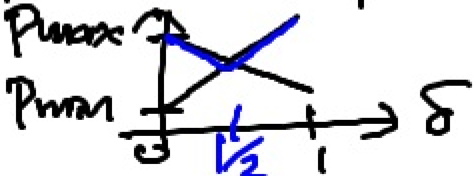
(a) Formulate

δ : fraction of ramen bought @ Homeplus

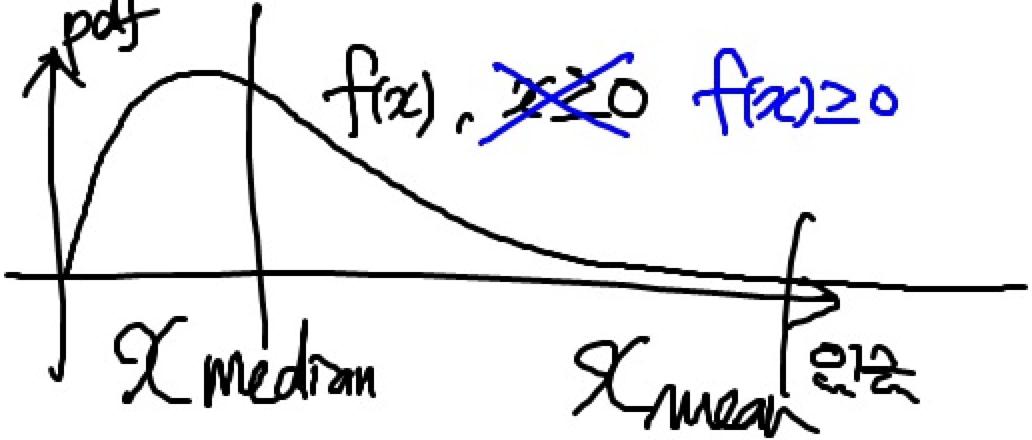
	pmin @ Homeplus	pmin @ Local
buy @ Homeplus	Pmin	Pmax
buy @ Local	Pmax	Pmin

$$\delta_{opt} \triangleq \min_{0 \le \delta \le 1} \left[\max \left\{ p_{min}\delta + p_{max}(1-\delta),\ p_{max}\delta + p_{min}(1-\delta) \right\} \right]$$

(b) $\delta_{opt} = \frac{1}{2}$ (c) $\frac{P_{min}+P_{max}}{2}$



p.2



$f(x),\ \cancel{x \geq 0}\ \ f(x) \geq 0$

$$x_{median} \triangleq \cancel{\frac{1}{2} =} \int_{-\infty}^{x_{median}} f(x)\,dx = \int_{x_{median}}^{+\infty} f(x)\,dx$$

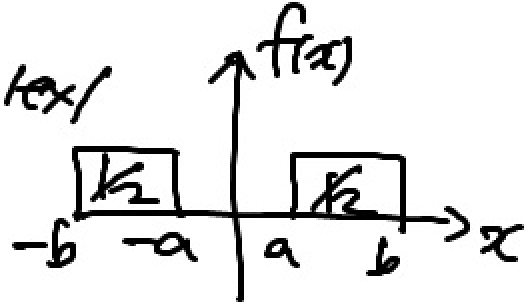
existence

uniqueness

★ Given $\{\underline{Y} = \underline{y}_0\}$

$$\int_{-\infty}^{\hat{\theta}} f_{\Theta|\underline{Y}}(\theta|\underline{y}_0)\,d\theta = \int_{\hat{\theta}}^{\infty} f_{\Theta|\underline{Y}}(\theta|\underline{y}_0)\,d\theta.$$

ex)



p3

$$\hat{\theta}_{MMSE}(\underline{y}) = \arg\min_{\hat{\theta}(\underline{y})} E\left[(\hat{\theta}(\underline{Y}) - \Theta)^2\right] = \arg\min_{\hat{\theta}} E\left[(\hat{\theta} - \Theta)^2 \mid \underline{Y} = \underline{y}\right]$$

$$g(\hat{\theta}) \triangleq \int_{-\infty}^{\infty} (\hat{\theta} - \theta)^2 f_{\Theta|\underline{Y}}(\theta|\underline{y})\, d\theta$$

$$\left.\frac{dg(\hat{\theta})}{d\hat{\theta}}\right|_{\hat{\theta} = \hat{\theta}_{opt}} = 0. \qquad \cancel{2}\int_{-\infty}^{\infty} (\hat{\theta}_{opt} - \theta) f_{\Theta|\underline{Y}}(\theta|\underline{y})\, d\theta = 0$$

$$\hat{\theta}_{opt} = \int_{-\infty}^{\infty} \theta f_{\Theta|\underline{Y}}(\theta|\underline{y})\, d\theta \triangleq E[\Theta \mid \underline{Y} = \underline{y}]$$

Belief.

Too good to be true!

$$\hat{\theta}_{MMSE}(\underline{y}) = E\{\underline{\Theta} \mid \underline{Y} = \underline{y}\}.$$