Lec. 15

Announcements

1. Quiz#3 on Next Friday
2. Correction
   - $E[\mathrm{Cov}(\underline{\Theta}|\underline{Y})] \neq \mathrm{Cov}(\underline{\Theta})$ in general even when $\underline{\Theta}$ & $\underline{Y}$ are jointly Gaussian.
   - When jointly Gaussian,

$$\mathrm{Cov}(\underline{\Theta}|\underline{Y}=\underline{y}) = \mathrm{Cov}(\underline{\Theta}|\underline{Y}=\underline{y}'),\ \forall y \neq y'.$$
$$\neq \mathrm{Cov}(\underline{\Theta})$$

Review

$$\underline{\hat{\theta}}_{MMSE}(\underline{y}) \triangleq \arg\min_{\underline{\hat{\theta}}(\underline{y})} \overset{\text{mean}}{E}\left\{\left\|\overbrace{\underline{\hat{\theta}}(\underline{Y})-\underline{\Theta}}^{\text{error}}\right\|^{\overset{\text{squared}}{2}}\right\}$$

$$\Rightarrow \underline{\hat{\theta}}_{MMSE}(\underline{y}) = E[\underline{\Theta} \mid \underline{Y}=\underline{y}]$$

$$MMSE = tr\{E[Cov(\underline{\Theta}\mid\underline{Y})]\}$$

Q. What if $\min_{\underline{\hat{\theta}}(\underline{y})} E\{(\underline{\hat{\theta}}(\underline{Y})-\underline{\Theta})^T \underbrace{C}_{>0} (\underline{\hat{\theta}}(\underline{Y})-\underline{\Theta})\}$

A. $\underline{\hat{\theta}}(\underline{y}) = \underline{\hat{\theta}}_{MMSE}(\underline{y})$, $\forall C>0$, $MMGSE = tr\{C\, E[Cov(\underline{\Theta}\mid\underline{Y})]\}$

p.3

Simplify!

Θ, Y are both scalar!

Unit

$$\begin{bmatrix} \Theta \\ Y \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_\Theta \\ \mu_Y \end{bmatrix}, \begin{bmatrix} \sigma_\Theta^2 & \rho\sigma_\Theta\sigma_Y \\ \rho\sigma_\Theta\sigma_Y & \sigma_Y^2 \end{bmatrix} \right)$$

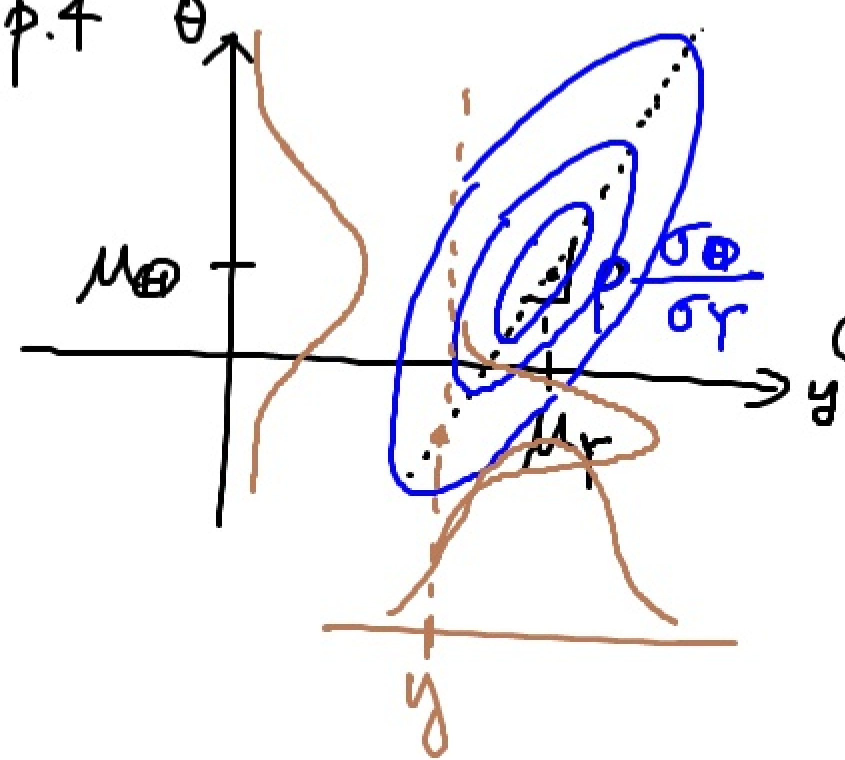
$$\rho \triangleq \frac{\mathrm{cov}(\Theta, Y)}{\sigma_\Theta \sigma_Y}$$

$$-1 \le \rho \le 1$$

$$E[(\Theta + xY)^2] \ge 0, \ \forall x$$

$e^{\boxed{}}$ must be unitless!

$$\begin{cases} e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \\ \ln x \end{cases}$$



$$E[\Theta \mid Y=y] = E[\Theta] + Cov(\Theta, Y)\,Cov(Y)^{-1}\,(y - E[Y])$$

$$= \mu_\Theta + \rho \frac{\sigma_\Theta}{\sigma_Y}(y - \mu_Y)$$

$$Cov(\Theta \mid Y=y) = Cov(\Theta) - Cov(\Theta, Y)\,Cov(Y)^{-1}\,Cov(Y, \Theta)$$

$$= \sigma_\Theta^2 - \rho^2 \sigma_\Theta^2$$

$$= (1-\rho^2)\,\sigma_\Theta^2$$