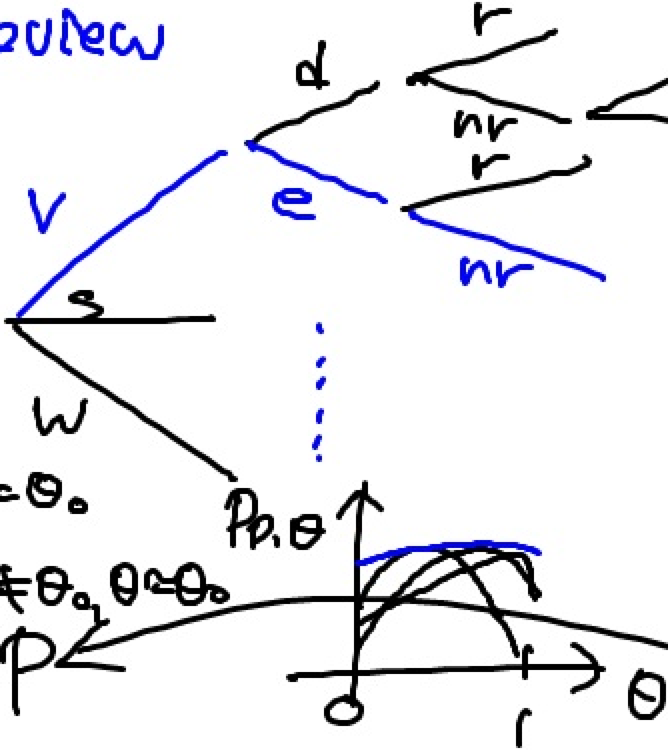
Lec. 17

Announcements

1. No class on this Thursday 😉.
2. Will make-up on Next(?) Friday 🙂.

Review



minimax

Neyman-Pearson

$\begin{cases} \max P_D \\ \text{s.t. } P_{FA} \le \alpha \end{cases}$

Composite hypothesis testing

$\updownarrow$

Simple

the nuisance parameter

$H_0: \theta = \theta_0$
vs
$H_1: \theta \neq \theta_0, \theta \approx \theta_0$

LMP

$\max \; P_{D,\underline{\theta}}$

s.t. $P_{FA,\underline{\theta}} \le \alpha, \; \forall \underline{\theta}$

the UMP detector!

p.2 Estimation error vector.

$$\hat{\underline{\theta}}(\underline{Y}) - \underline{\theta}$$

cf) $\underset{\underline{Y},\underline{\Theta}}{E}\{\|\hat{\underline{\theta}}(\underline{Y}) - \underline{\Theta}\|^2\}$

in random parameter estimation.

$$\underset{\underline{Y}}{E}\{\|\hat{\underline{\theta}}(\underline{Y}) - \underline{\theta}\|^2\}$$

← a function of $\underline{\theta}$

simplify

visualize

$$\underline{x} = \begin{bmatrix} -1 \\ 2 \\ -3 \\ 0 \end{bmatrix}$$

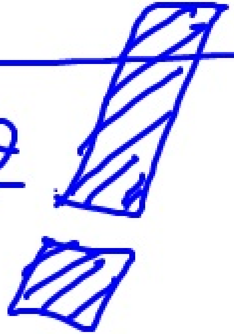
$\|\underline{x}\|_0 = 3$

$\|\underline{x}\|_1 = 6$

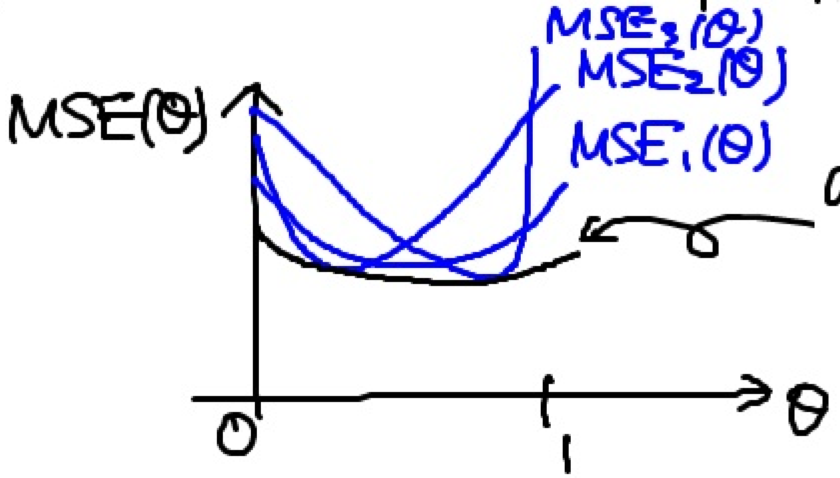
$\|\underline{x}\|_2 = \sqrt{14}$

$\|\underline{x}\|_\infty = 3$

$$\|\underline{x}\|_n = \left(\sum_i |x_i|^n\right)^{\frac{1}{n}}$$



p.3 MSE as a function of θ



$$\arg\min_{\underline{\hat{\theta}}(\underline{y})} \ \underset{\underline{Y}}{E}\{\|\underline{\hat{\theta}}(\underline{Y})-\underline{\theta}\|^2\}, \forall \underline{\theta}$$

$$\text{s.t.} \quad \underset{\underline{Y}}{E}\{\underline{\hat{\theta}}(\underline{Y})-\underline{\theta}\}=\underline{0}, \quad \forall \underline{\theta}.$$

In general, no such $\underline{\hat{\theta}}(\underline{y})$ exists.

We need a meaningful constraint that may result in the existence of an optimal solution!

P4

$\underline{b}(\underline{\theta}) \triangleq \underset{\underline{Y}}{E}\{\hat{\underline{\theta}}(\underline{Y})-\underline{\theta}\}$: the bias of the estimator $\hat{\underline{\theta}}(\underline{y})$.

$\underline{b}(\underline{\theta}) = \underline{0}, \forall \underline{\theta}$: unbiased.

$$\min_{\hat{\underline{\theta}}(\underline{y})} \underset{\underline{Y}}{E}\{\|\hat{\underline{\theta}}(\underline{Y})-\underline{\theta}\|^2\}, \forall \underline{\theta}$$

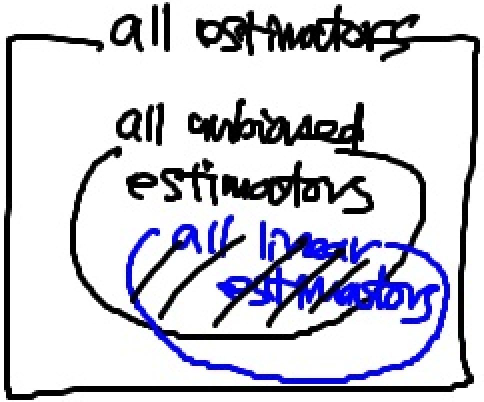
$$\text{s.t.} \quad \underset{\underline{Y}}{E}\{\hat{\underline{\theta}}(\underline{Y})-\underline{\theta}\} = \underline{0}, \forall \underline{\theta}$$

minimum M
variance V
unbiased U
estimator E

the set of all unbiased estimators $= \{\hat{\underline{\theta}}(\underline{y}) : \underline{b}(\underline{\theta}) = \underline{0}, \forall \underline{\theta}\}$.

p.5

MVUE



B best
L linear
U unbiased
E estimator

"non-random parameter estimator problems":

$$\left\{ f_{\underline{Y};\underline{\theta}}(\underline{y}) \; : \; \underline{\theta} \in A \right\}$$

an uncountable set.

a family of pdf's of $\underline{Y}$, parameterized by $\underline{\theta}$.