Lec. 18

Announcements

1. No class on this Thursday
2. Make-up classes on this Friday.
3. 2nd mid-term exam on 11/15.

Review

- MMSE $\longrightarrow$ orthogonality principle

  AMMSE

  LMMSE

  sequence obser.

Generalize

$\updownarrow$

Specialize

- Best Linear Unbiased Estimator (BLUE)

p2

MVUE (minimum variance unbiased estimator)

UMVU (uniformly minimum variance unbiased) estimator

cf) UMP detector

\+ linearity

BLUE

\- existence ← necessary

uniqueness ← optional.

p3

$$\min_{\underline{w}} \quad \underline{w}^T R \underline{w}$$

$$\text{s.t.} \quad \underline{w}^T \underline{x} = 1$$

$\Rightarrow$ Lagrange multiplier theorem

$R = R^T > 0$

$$\Rightarrow \begin{cases} \dfrac{\partial L}{\partial \underline{w}} = 2R\underline{w} - \lambda \underline{x} = 0 \\ \dfrac{\partial L}{\partial \lambda} = 1 - \underline{w}^T \underline{x} \end{cases}$$

$$\Rightarrow \quad \underline{w}_{opt} = \frac{\lambda}{2} R^{-1} \underline{x}$$

Step 1. Construct the Lagrangian function

$$L(\underline{w}, \lambda) = \underline{w}^T R \underline{w} + \lambda (1 - \underline{w}^T \underline{x})$$

Step 2.

$$\left.\begin{aligned} \left.\frac{\partial L}{\partial \underline{w}}\right|_{\underline{w}=\underline{w}_{opt},\, \lambda=\lambda_{opt}} &= \underline{0} \\ \left.\frac{\partial L}{\partial \lambda}\right|_{\underline{w}=\underline{w}_{opt},\, \lambda=\lambda_{opt}} &= 0 \end{aligned}\right\} \text{ simultaneous equation.}$$

p.4

$$\begin{pmatrix} \dfrac{\partial L}{\partial \underline{w}} = 2R\underline{w} - \lambda \underline{x} \\[2ex] \dfrac{\partial L}{\partial \lambda} = 1 - \underline{w}^T \underline{x} \end{pmatrix} \Rightarrow \begin{pmatrix} 2R\underline{w}_{opt} - \lambda_{opt} \underline{x} = \underline{0} \\[2ex] 1 - \underline{w}_{opt}^T \underline{x} = 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \underline{w}_{opt} = \dfrac{\lambda_{opt}}{2} R^{-1} \underline{x} \\[2ex] \dfrac{\lambda_{opt}}{2} \underline{x}^T R^{-1} \underline{x} = 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda_{opt} = \dfrac{2}{\underline{x}^T R^{-1} \underline{x}} \\[2ex] \underline{w}_{opt} = \dfrac{R^{-1} \underline{x}}{\underline{x}^T R^{-1} \underline{x}} \end{pmatrix}$$

$$\therefore \hat{\theta}_{BLUE}(\underline{y}) = \frac{\underline{x}^T R^{-1} \underline{y}}{\underline{x}^T R^{-1} \underline{x}}$$

p.5

a linear estimator.

From $\underline{Y} = \underline{x}\theta + N$, we know that

$\frac{Y_1}{x_1}$ is an unbiased estimator.

$$E_\theta\left\{\left(\frac{Y_1}{x_1} - \theta\right)^2\right\} = E_\theta\left\{\left(\frac{x_1\theta + N_1}{x_1} - \theta\right)^2\right\}$$

$$= \frac{[R]_{1,1}}{x_1^2} \qquad , \qquad \frac{[R]_{2,2}}{x_2^2}$$

Inequalities!

BLUE : $\frac{1}{\underline{x}^T R^{-1} \underline{x}}$