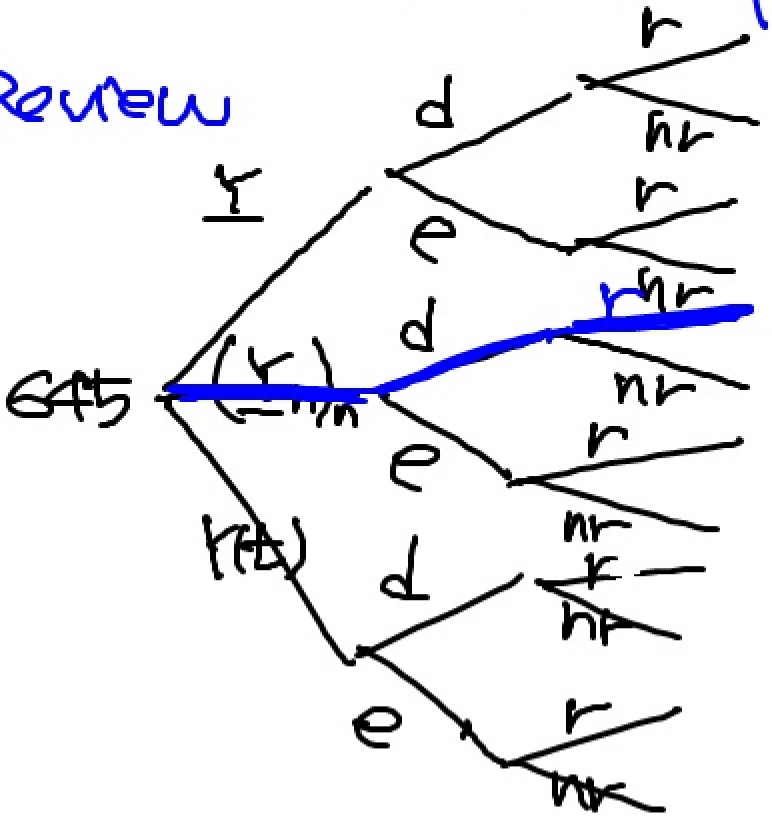
Lec. 29

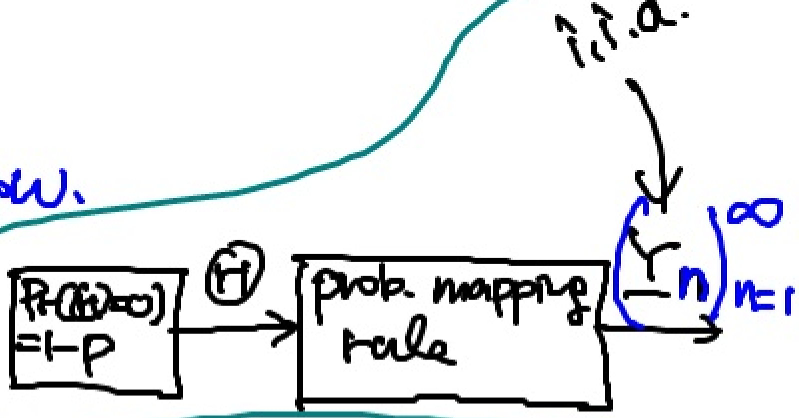
Announcement: Make-up tomorrow.

Review



Wiener, Kalman

MLE



$$\{ f_{\underline{Y}_n|\Theta=1}(\underline{y}_n),\ f_{\underline{Y}_n|\Theta=0}(\underline{y}_n) \}$$

e.g.

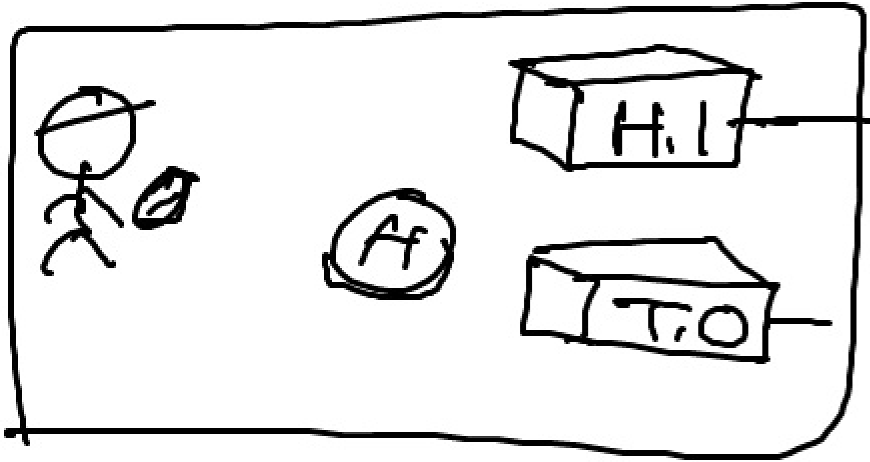
$$H_0 : \underline{Y}_n \sim N(\underline{0}, \sigma^2 I)$$

vs

$$H_1 : \underline{Y}_n \sim N(\mu \underline{1}, \sigma^2 I)$$

$\mu \neq 0.$

p. 2



$(\underline{r}_n)_{n=1}^{\infty}$

Θ

$\theta \in \{0, 1\}$

$\Theta \in \{H, T\}$

$\Pr(\Theta = 0) = 1-p$

$\Pr(\Theta = H) = p$

$\Pr(\Theta = 1) = p$

$\Pr(\Theta = 0) = 1-p$ $\quad p$ known

$H_0 : \underline{r} \sim N(\underline{0}, \sigma^2 I)$

vs

$H_1 : \underline{r} \sim N(\mu \underline{1}, \sigma^2 I)$

$c_{\hat{\theta}, \theta}$

p.3

If we observe the entire sequence,

then $E[C] = E\{E[C \mid \Theta]\}$

$= E[C \mid \Theta = 0] Pr(\Theta = 0) +$

$E[C \mid \Theta = 1] Pr(\Theta = 1)$

$= C_{0,0} Pr(\Theta = 0) + C_{1,1} Pr(\Theta = 1)$

$P(\Theta = 0) = 1 - p$

$Pr(\Theta = 1) = p$

$\left\{ \begin{array}{l} C_{\hat{\theta},\theta} \\ C \end{array} \right.$ ← cost per observation.

pf

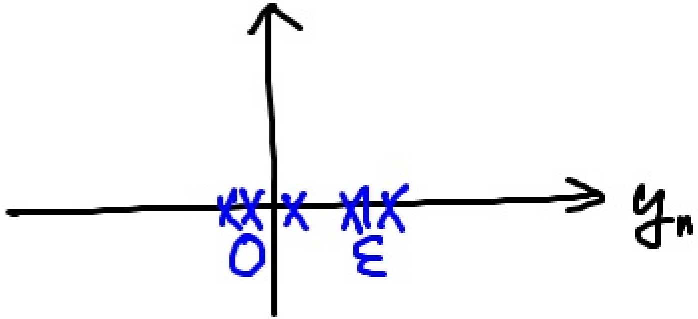
$\Pr(\Theta=1)=p$

$\Pr(\Theta=0)=1-p$

$H_0: Y_n \sim N(0,\sigma^2)$

vs

$H_1: Y_n \sim N(\varepsilon,\sigma^2)$. $\varepsilon>0$



Every science/engineering discovery starts from

observation.

$$\delta(\cdot)=\begin{cases} GO, \\ 0, \\ 1, \end{cases}$$

No observations

$$\delta(Y_1)=\begin{cases} GO, \\ 0, \\ 1, \end{cases}$$

$$\delta(Y_1,Y_2)=\begin{cases} GO \\ 0, \\ 1, \end{cases}$$

$\vdots$

P5

$$\delta \begin{cases} \phi & \text{"go"} \quad \text{"stop"} \\ \hat{\theta} & 0,\ 1 \end{cases}$$

$\delta_0 = (\phi_0, \hat{\theta}_0)$ a <u>sequence</u> of ⟨stopping rules⟩

&

$\delta_1(\underline{y}_1) = \left(\phi_1(\underline{y}_1), \hat{\theta}_1(\underline{y}_1)\right)$ a ____"____ " ⟨decision rules⟩

$\vdots$

$\delta_n(\underline{y}_1, \cdots, \underline{y}_n) = \left(\phi_n(\underline{y}_1, \cdots, \underline{y}_n), \hat{\theta}_n(\underline{y}_1, \cdots, \underline{y}_n)\right)$

$\vdots$

p.6

uniform cost.

$c > 0$.

$\delta_0 = (\phi_0, \hat{\theta}_0) = (S, 0) \Rightarrow E[C] = 1 \times \Pr(\Theta = 1) = p$

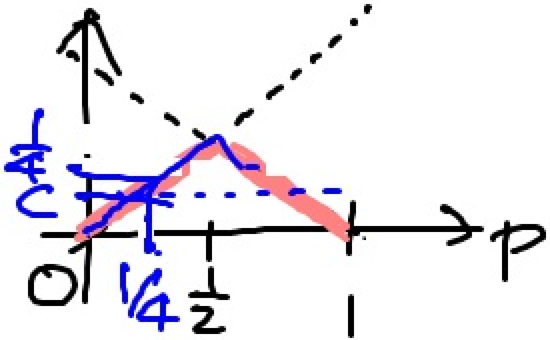
$\delta_0 = (\phi_0, \hat{\theta}_0) = (S, 1) \Rightarrow E[C] = 1 \times \Pr(\Theta = 0) = 1-p$.

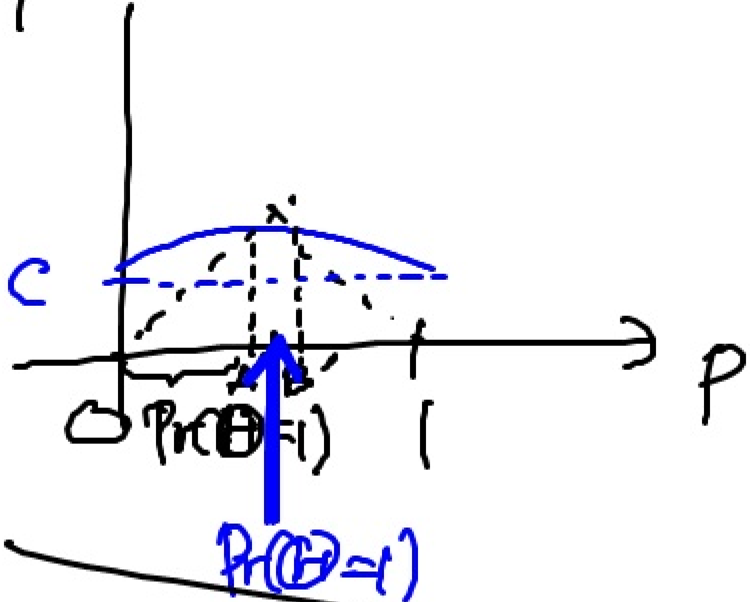
$p = \frac{1}{2}$, $c > \frac{1}{2}$.

$\cancel{p \neq \frac{1}{2}}$ $p = \frac{1}{4}$, $c > \frac{1}{3}$

$p =$

Visualize





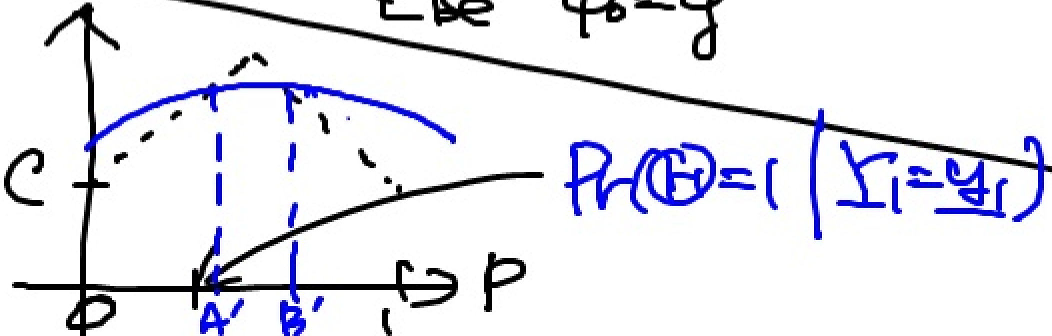
$\delta_0 = (\phi_0, \hat{\theta}_0)$

If $Pr(\Theta=1) < A'$ or
$Pr(\Theta=1) > B'$

$\phi_0 = S$, $\hat{\theta}_0 = 0$ $\hat{\theta}_0 = 1$

Else $\phi_0 = g$

$\delta_1 = (\phi_1, \hat{\theta}_1)$



p 8

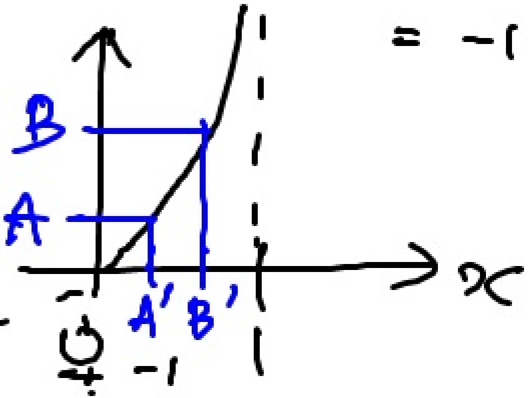
SPRT(A, B)

sequential
probability
ratio
test
w/ thresholds A & B

$$\Pr(\Theta=1 \mid Y_1=y_1, \dots, Y_n=y_n) \triangleq \pi_{1,n}$$

$$\pi_{1,0} = \Pr(\Theta=1)$$

$$\frac{x}{1-x} = \frac{1-1+x}{1-x} = -1 + \frac{1}{1-x}$$



$$1-\pi_{1,n} \triangleq \pi_{0,n} = \Pr(\Theta=0 \mid Y_1=y_1, \dots, Y_n=y_n)$$

p. 9

$$\frac{\pi_{1,n}}{1-\pi_{1,n}} = \frac{\pi_{1,n}}{\pi_{0,n}} = \left(\frac{\pi_{1,0}}{\pi_{0,0}}\right)\left(\frac{f_{\underline{Y}_1|\theta=1}(\underline{y}_1)}{f_{\underline{Y}_1|\theta=0}(\underline{y}_1)}\right)\cdots\left(\frac{f_{\underline{Y}_n|\theta=1}(\underline{y}_n)}{f_{\underline{Y}_n|\theta=0}(\underline{y}_n)}\right)$$

$$= \frac{\pi_{1,n-1}}{\pi_{0,n-1}} \; \frac{f_{\underline{Y}_n|\theta=1}(\underline{y}_n)}{f_{\underline{Y}_n|\theta=0}(\underline{y}_n)}$$

$C_{\theta,\theta}$: uniform cost

$C>0$: cost/observation

A, B.

SPRT(A,B) 中, optimal detector exists.