Lec. 32

Announcements

1. Quiz #5 tomorrow

2. A make-up on Monday

Review

observable   nonobservable

Lec. 31	Lec. 32
$(\underline{Y}[n])_n$, $(\underline{X}[n])_n$, LMMSE	$(\underline{Y}[n])_n$, $(\underline{X}[n])_n$, AMMSE
jointly WSS / zero-mean	nonstationary
$R_{\underline{X}\underline{X}}[n-m]$ $R_{\underline{Y}\underline{Y}}[n-m]$ $R_{\underline{X}\underline{Y}}[n-m]$.	
Wiener filtering	Kalman filtering

p.2

Review of AMMSE estimation

Given $\underline{\Theta}, \underline{Y}$ jointly Gaussian $\{\underline{Y}=\underline{y}\}$

$$E[\underline{\Theta} | \underline{Y}=\underline{y}] = E[\underline{\Theta}] + Cov(\underline{\Theta},\underline{Y})\, Cov(\underline{Y})^{-1}(\underline{y}-E[\underline{Y}])$$

$$Cov[\underline{\Theta} | \underline{Y}=\underline{y}] = Cov(\underline{\Theta}) - Cov(\underline{\Theta},\underline{Y})\, Cov(\underline{Y})^{-1} Cov(\underline{Y},\underline{\Theta})$$

Given $\underline{X}, \underline{Y}$ jointly random $\{\underline{Y}=\underline{y}\}$

$$\hat{\underline{X}}_{AMMSE}(\underline{y}) = E[\underline{X}] + Cov(\underline{X},\underline{Y})\, Cov(\underline{Y})^{-1}(\underline{y}-E[\underline{Y}])$$

$$E[(\hat{\underline{X}}_{AMMSE}(\underline{Y})-\underline{X})(\hat{\underline{X}}_{AMMSE}(\underline{Y})-\underline{X})^T]$$

$$= Cov(\underline{X}) - Cov(\underline{X},\underline{Y})\, Cov(\underline{Y})^{-1} Cov(\underline{Y},\underline{X})$$

p.3 Assume $(X[n])_n$ & $(Y[n])_n$ are jointly Gaussian random processes. $\{ Y[n] = \underline{y}[n],\ Y[n-1] = \underline{y}[n-1], \cdots \}$

$$E[X[n] \mid Y[n] = \underline{y}[n],\ Y[n-1] = \underline{y}[n-1], \cdots]$$

$=$ in terms of $E[X[n] \mid Y[n-1] = \underline{y}[n-1],\ Y[n-2] = \underline{y}[n-2] \cdots]$, $\cdots$, $\underline{y}[n]$.

Given jointly Gaussian X, Y, and Z,

$E[X \mid Y = \underline{y},\ Z = \underline{z}] =$

$Cov[X \mid Y = \underline{y},\ Z = \underline{z}] =$

$$P(A \mid B \cap C) = \frac{P(A \cap B \mid C)}{P(B \mid C)}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

p4

$$E[\underline{X} \mid \underline{Y}=\underline{y}, \underline{Z}=\underline{z}] = E[\underline{X} \mid \underline{Z}=\underline{z}] + \mathrm{Cov}(\underline{X},\underline{Y} \mid \underline{Z}=\underline{z})\, \mathrm{Cov}(\underline{Y} \mid \underline{Z}=\underline{z})^{-1} \left(\underline{y} - E[\underline{Y} \mid \underline{Z}=\underline{z}] \right)$$

$$\mathrm{Cov}[\underline{X} \mid \underline{Y}=\underline{y}, \underline{Z}=\underline{z}] = \mathrm{Cov}(\underline{X} \mid \underline{Z}=\underline{z}) - \mathrm{Cov}(\underline{X},\underline{Y} \mid \underline{Z}=\underline{z})\, \mathrm{Cov}(\underline{Y} \mid \underline{Z}=\underline{z})^{-1}\, \mathrm{Cov}(\underline{Y},\underline{X} \mid \underline{Z}=\underline{z})$$

$\underline{X} \rightarrow X[n]$

$\underline{Y} \rightarrow Y[n]$

$\underline{Z} \rightarrow Y[n-1], Y[n-2] \ldots$

$E[\underline{X} \mid \underline{Z}=\underline{z}] \rightarrow \hat{X}_{n|n-1}$

$E[\underline{X} \mid \underline{Y}=\underline{y}, \underline{Z}=\underline{z}] \rightarrow \hat{X}_{n|n}$

$\mathrm{Cov}[\underline{X} \mid \underline{Y}=\underline{y}, \underline{Z}=\underline{z}] = \Sigma_{n|n}$

$\mathrm{Cov}[\underline{X} \mid \underline{Z}=\underline{z}] = \Sigma_{n|n-1}$

p.5

$$\mathrm{Cov}\left(X[n], Y[n] \mid Y[n-1]=y[n-1], \cdots \right)$$

$$= E\Big[\big(X[n] - E(X[n] \mid Y[n-1]=y[n-1], Y[n-2]=y[n-2], \ldots)\big)$$

$$\big(Y[n] - E(Y[n] \mid Y[n-1]=y[n-1], Y[n-2]=y[n-2], \ldots)\big) \mid$$

$$Y[n-1]=y[n-1], Y[n-2]=y[n-2], \ldots \Big]$$

$$= \mathrm{Cov}\left(X[n], X[n] \mid Y[n-1]=y[n-1], \cdots \right) C_n^T \qquad Y[n] = C_n X[n] + V[n]$$

$$\Rightarrow \Sigma_{n|n-1} C_n^T$$

$$X[n+1] = A_n X[n] + B_n u[n]$$

$$\hat{X}_{n+1|n} \Rightarrow E\{X[n+1] \mid Y[n]=y[n], Y[n-1]=y[n-1] \ldots\}$$

$$A_n \hat{X}_{n|n} \Leftarrow A_n E[X[n] \mid \quad " \quad " \quad]$$

p.17

$$\hat{x}_{n+1|n} = A_n \hat{x}_{n|n-1} + A_n \Sigma_{n|n-1} C_n^T (C_n \Sigma_{n|n-1} C_n^T + R_n)^{-1} (y_n - C_n \hat{x}_{n|n-1})$$

$$\begin{cases} \hat{x}_{n|n} = \\ \hat{x}_{n+1|n} = A_n \hat{x}_{n|n} \end{cases}$$

E

$$E[X] = E(E(X|Y))$$

$$E[X|Z=z] = E[E[X|Y,Z]|Z=z]$$