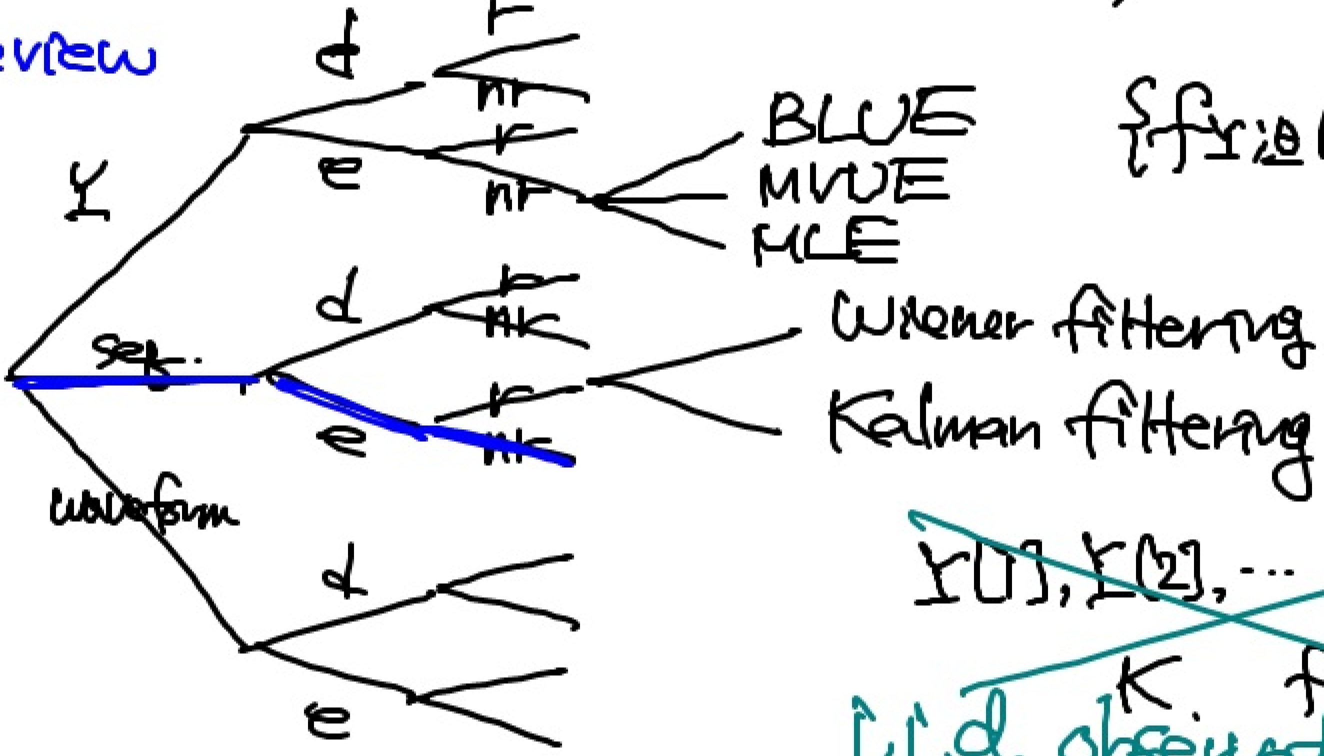
Lec. 33

Announcement Quiz #6 this Friday

Review



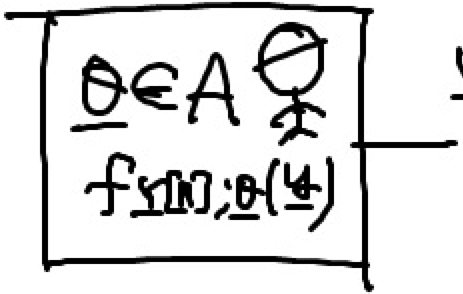
$\{f_{\underline{Y};\underline{\theta}}(\underline{y}) : \underline{\theta} \in \Lambda\}$

$\cancel{\underline{Y}[1], \underline{Y}[2], \ldots}$

$\cancel{K. \quad f_{\underline{Y}[1],\underline{Y}[2],\underline{Y}[3\ldots]}\ldots}$

i.i.d. observations?

p.2



$\underline{Y}[1], \underline{Y}[2], \cdots$ i.i.d.

independent and identically distributed

$$\hat{\underline{\theta}}_1(\underline{y}_1)$$

sequential estimator

$$\hat{\underline{\theta}}_2(\underline{y}_1, \underline{y}_2)$$

(unbiased
mini cov. matrix

$$\vdots$$

$$\hat{\underline{\theta}}_k(\underline{y}_1, \underline{y}_2, \cdots, \underline{y}_k) = \hat{\underline{\theta}}_k(\underline{y}^{(k)})$$

$$\vdots$$

p.3 $\underbrace{\hat{\underline{\theta}}_k(\underline{Y}^{(k)})}_{\text{random seq.}} \xrightarrow{k\to\infty} \underline{\theta}, \quad \forall \underline{\theta}$

real analysis.
해석학

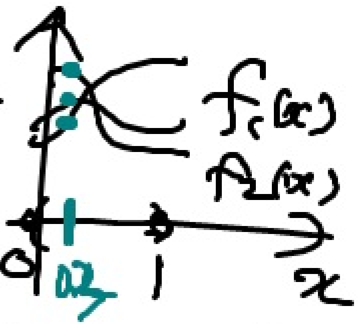
Def. A seq. of real numbers $(x_k)_{k\ge 1}$ converges to x_0

if $\quad \forall \varepsilon>0, \exists N(\varepsilon)\in\mathbb{N}$ s.t.

$$k\ge N(\varepsilon) \Rightarrow |x_k-x_0|\le\varepsilon$$

ε-δ 法

continuity of a functn



Def. A seq. of real functions $(f_k(x))_{k\ge 1}$ converge to $f_0(x)$ if ....

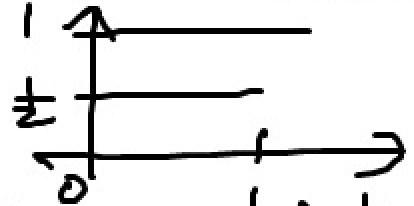
p 4 Def. (uniform convergence)

UC
$\Downarrow$
e
$\Downarrow$
a.e. $\searrow$ in measure $\swarrow$ L^p

Def. (convergence everywhere)

$$f_k(x) \to f_0(x), \quad \forall x \in \Omega$$

$$\Omega = \{x \in \Omega : f_k(x) \to f_0(x)\}$$



$$f_k(x) = \begin{cases} 1-\left(\frac{1}{2}\right)^k, & 0<x<1, \ \notin \mathbb{Q} \\ 0 & \in \mathbb{Q} \end{cases}$$

$$f_0(x) = 1, \quad 0<x<1$$

Def. (Convergence almost everywhere)

$$m(\{x \in \Omega : f_k(x) \not\to f_0(x)\}) = 0.$$

Def. $\int_\Omega |f_k(x) - f_0(x)|^p \, dx \longrightarrow 0$ as $k \to \infty$. (Convergence in L^p) $1 \le p < \infty$

p5

Def (Convergence in measure)

$$\forall \varepsilon>0,\quad m\left(\{x\in\Omega : |f_k(x)-f_0(x)|>\varepsilon\}\right)\to 0 \text{ as } k\to\infty$$

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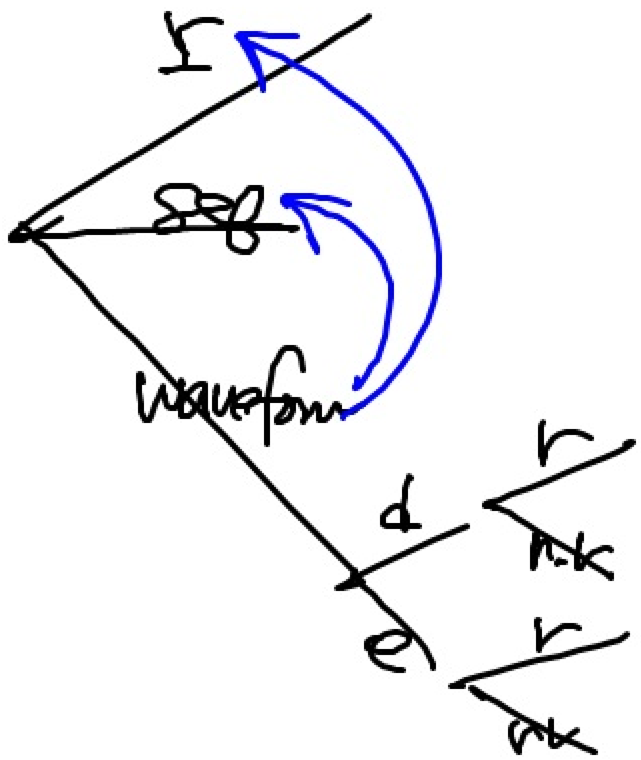
modes of convergence of a seq. of functions.

$$\begin{array}{ccc}
\text{UC} & & \\
\Downarrow & & \\
\text{e} & & \\
\Downarrow & & \\
\text{a.e.} & & L^p \\
 & \searrow \quad \swarrow & \\
 & \text{in measure.} &
\end{array}$$

modes of convergence of a seq. of random variables.

$$\begin{array}{ccc}
\overset{\text{SLLN}}{\text{a.s.}} & & L^p \\
 & \searrow \quad \swarrow & \\
 & \text{in prob. WLLN} & \\
 & \Downarrow & \\
 & \text{in d. CLT.} &
\end{array}$$

p.6



KL expansion.