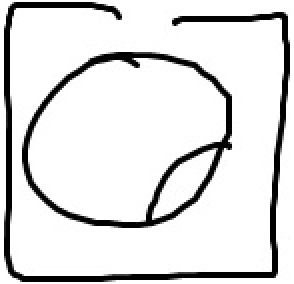
Lec. 36

Announcement

Make-up tomorrow



cf) DFT = DTFS

$p(t),\ 0 \le t < T$

$\tilde{p}(t) = \sum_{n=-\infty}^{\infty} p(t-nT)$

CTFS

Review

	Deterministic CT	Random CT
TL	periodic extension MTD CTFS.	KL expansion
BL	periodic 〃 MTD CTFS. Nyquist ⟹ sampling theorem	⟹ (Shannon)-Nyquist sampling theorem
SI	→ ⋮	Cyclostationary r.p.

p.2

the Fredholm integral equation

TD random CT observation

— $Y(t),\ 0 \le t < T$

$\uparrow$ 2nd order r.p.

$E[Y(t)] = \mu(t),\ 0 \le t < T$

$Cov(Y(t)) \triangleq E[(Y(t)-\mu(t))(Y(u)-\mu(u))]$

$= C_{YY}(t,u),\ 0 \le t,u < T$

eigenvalue

— $\int_0^T C_{YY}(t,u)\psi_k(u)du = \lambda_k \psi_k(t),\ 0 \le t < T$

p.s.d. Kernel

eigenfunction of $C_{YY}(t,u)$

p.3

— Find

$(\lambda_k, \phi_k(t))_{k=1}^{\infty}$ s.t.

$$\int_0^T \phi_k(t)\phi_{k'}(t)\,dt = \delta_{k,k'}$$

$\lambda_k > 0$

— $$C_{XX}(t,u) = \sum_{k=1}^{\infty} \lambda_k \phi_k(t)\phi_k(u), \quad 0 \le t, u < T.$$

Mercer's theorem.

— $$Y(t) = \sum_{k=1}^{\infty} Y_k \phi_k(t), \quad 0 \le t < T$$

the KL expansion

$Y_k \triangleq \int_0^T Y(t)\phi_k(t)\,dt$

$\mathrm{Var}(Y_k) = \lambda_k$

$E[Y_k] = \int_0^T \mu(t)\phi_k(t)\,dt$

p. 4

Grenander's theorem

the Radon-Nikodym derivative

Example "Likelihood ratio is not well defined."

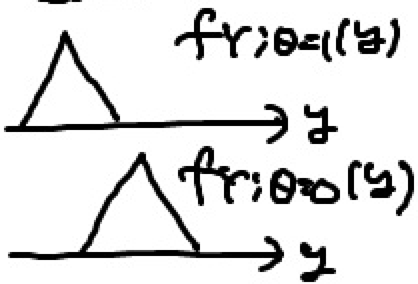
$H_0 : \underline{Y} \sim f_{\underline{Y};\theta=0}(\underline{y})$

vs

$H_1 : \underline{Y} \sim f_{\underline{Y};\theta=1}(\underline{y})$

$$\frac{f_{\underline{Y};\theta=1}(\underline{y})}{f_{\underline{Y};\theta=0}(\underline{y})} \begin{cases} = \infty & \underline{y} \in S_1 \setminus S_0 \\ < \infty & \underline{y} \in S_1 \cap S_0 \\ = 0 & \underline{y} \in S_0 \setminus S_1 \end{cases}$$

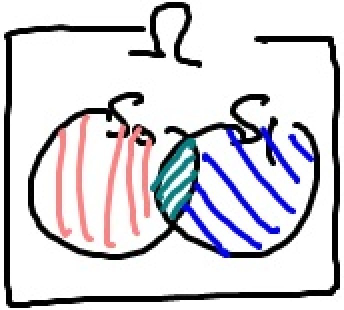
Extreme case



singular detection

$S_1 \leftarrow$ support of

$S_0 \leftarrow$ $f_{\underline{Y};\theta=1}(\underline{y})$



p5

$$f(\underline{y}) \triangleq \frac{f_{\underline{Y};\theta=1}(\underline{y})}{f_{\underline{Y};\theta=0}(\underline{y})} \quad \text{for } \underline{y} \in S_0$$

$$f_{\underline{Y};\theta=1}(\underline{y}) = f(\underline{y})\, f_{\underline{Y};\theta=0}(\underline{y})$$

~ the Radon-Nikodym derivative

$$Y(t) \equiv (Y_k)_{k=1}^{\infty}$$

$\underline{\theta} \in A$

Y_1 $\quad f_{Y_1;\theta}(y_1)$

(Y_1, Y_2) $\quad f_{Y_1,Y_2;\theta}(y_1, y_2)$

(Y_1, Y_2, Y_3) $\quad f_{Y_1,Y_2,Y_3;\theta}(y_1, y_2, y_3)$

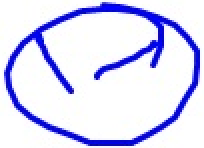
$\vdots$

$$\lim_{N\to\infty} f_{Y_1,Y_2,Y_3,\cdots,Y_N;\theta}(y_1,\cdots,y_N) = 0.$$

p6

$$A=\{0,1\},\quad \lim_{N\to\infty} \frac{f_{r_1,r_2,\cdots,r_N;\theta=1}(y_1,\cdots,y_N)}{f_{r_1,r_2,\cdots,r_N;\theta=0}(y_1,\cdots,y_N)} = f(y_1,\cdots)$$

$Q\,K_0$?



$H_0 : r(t) \sim P_0 \qquad (\Omega_0, \mathcal{F}_0, P_0)$

vs

$H_1 : r(t) \sim P_1 \qquad (\Omega_1, \mathcal{F}_1, P_1)$

For an event $F \in \mathcal{F}_1$,

$P_0(F)$

$$P_1(F) = \int_F f \, dP_0 + P_1(F \cap H)$$

$$dP_1 = f \, dP_0$$

↳ the Radon-Nikodym derivative