Lec. 38

Review

Detection
Estimation } in AWGN, $0 \le t < T$

Preview

Detection
Estimation } in AGN, $0 \le t < T$

Least Squares (LS) Estimation
Method of Moments Estimation
Spectral Estimation

p.2

H_1 : $r(t) = s_1(t) + N(t)$ → additive Gaussian noise

$E[N(t)] = 0$

vs

$E[N(t)N(s)] = R_{NN}(t,s) \succeq 0$

H_2 : $r(t) = s_2(t) + N(t)$

→ known deterministic

$P = U_1(U_1^T U_1)^{-1} U_1^T$

$= U_1 U_1^T$

cf)

H_1 : $\underline{r} = \underline{s}_1 + \underline{N} = P\underline{s}_1 + (I-P)\underline{s}_1 + \underline{N}$

$I - P$

$= U_2 U_2^T$

vs

H_2 : $\underline{r} = \underline{s}_2 + \underline{N} = P\underline{s}_2 + (I-P)\underline{s}_2 + \underline{N}$

$E[\underline{N}] = 0$

Singular Detection!

$$E[\underline{N}\,\underline{N}^T] = R_{NN} = [U_1 U_2]\begin{bmatrix} \lambda_1 & & & \\ & \ddots & & \\ & & & \\ & & & \lambda_M \end{bmatrix} U^T$$

p3

$$g_i(t) \triangleq \sum_{k=1}^{\infty} \frac{s_{i,k}}{\lambda_k} \varphi_k(t), \quad 0 \le t < T.$$

$$\int_0^T y(t) g_i(t)\,dt = \int_0^T \sum_{k=1}^{\infty} y_k \varphi_k(t) \sum_{k'=1}^{\infty} \frac{s_{i,k'}}{\lambda_{k'}} \varphi_{k'}(t)\,dt$$

$$= \sum_k \sum_{k'} y_k \frac{s_{i,k'}}{\lambda_{k'}} \underbrace{\int_0^T \varphi_k(t) \varphi_{k'}(t)\,dt}_{\delta_{k,k'}}$$

$$= \sum_{k=1}^{\infty} y_k \frac{s_{i,k}}{\lambda_k}$$

$$\int_0^T s_i(t) g_i(t)\,dt = \cdots = \sum_{k=1}^{\infty} s_{i,k} \frac{s_{i,k}}{\lambda_k}$$

p.4

$$H_0 : \underline{Y} = \underline{s}_0 + \underline{N} \qquad \underline{N} \sim N(\underline{0}, R)$$

vs

$$H_1 : \underline{Y} = \underline{s}_1 + \underline{N}$$

$$R = R^T \succ 0$$

$$= U \Lambda U^T \qquad UU^T = I$$

$$\Lambda = \mathrm{diag}(\cdot\ -)$$

$$R^{-\frac{1}{2}} \triangleq U \Lambda^{-\frac{1}{2}} U^T$$

$$\underline{Y} \longrightarrow \boxed{R^{-\frac{1}{2}}\underline{y}} \longrightarrow R^{-\frac{1}{2}}\underline{Y} = \underline{Z}$$

$$H_0 : \underline{Z} = R^{-\frac{1}{2}}\underline{s}_0 + \underline{W}$$

vs

$$H_1 : \underline{Z} = R^{-\frac{1}{2}}\underline{s}_1 + \underline{W}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix}$$

$$\Lambda^{-\frac{1}{2}} = \begin{bmatrix} \frac{1}{\sqrt{\lambda_1}} & & \\ & \frac{1}{\sqrt{\lambda_2}} & \\ & & \ddots \end{bmatrix}$$

$$\underline{W} = R^{-\frac{1}{2}}\underline{N} \sim N(\underline{0}, I)$$

$$E[\underline{W}\,\underline{W}^T] = E[R^{-\frac{1}{2}}\underline{N}\,\underline{N}^T R^{-\frac{1}{2}}] = R^{-\frac{1}{2}} R R^{-\frac{1}{2}} = I$$

p.5

H_0 : $r(t) = s_0(t) + N(t)$

vs.

H_1 : $r(t) = s_1(t) + N(t)$

$E[N(t)] = 0,$

$E[N(t)N(s)] = R_{NN}(t,s) > 0$

$$= \sum_{k=1}^{\infty} \lambda_k \phi_k(t) \phi_k(s)$$

cf) $R = U \Lambda U^T$

$= \sum \lambda_k \underline{u}_k \underline{u}_k^T$

$R^{-\frac{1}{2}} = \sum \frac{1}{\sqrt{\lambda_k}} \underline{u}_k \underline{u}_k^T$

$$h_w(t,s) = \sum_{k=1}^{\infty} \frac{1}{\sqrt{\lambda_k}} \phi_k(t) \phi_k(s), \quad 0 \le t,s \le T.$$

$$r(t) \longrightarrow \boxed{h_w(t,s)} \longrightarrow z(t)$$

Linear Time-varying whitening filter.

where $z(t) = \int_0^T h_w(t,s) r(s) ds$

$H_0 : z(t) = \tilde{s}_0(t) + W(t)$

$H_1 : z(t) = \tilde{s}_1(t) + W(t)$

p.6

Inference problem
- Observation $(V_i, I_i)_{i=1}^{N}$
- parameter to be inferred. R

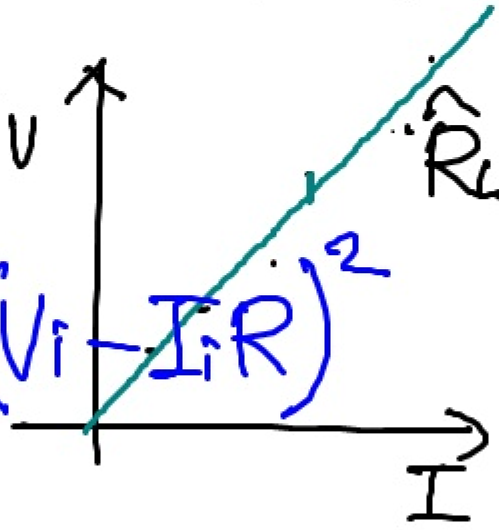
Q. Find R

Least Squares Estimate of R

Linear Regression

A. ① $V_i = I_i R$

$$\hat{R}_{LS} = \underset{R}{\operatorname{argmin}} \sum_{i=1}^{N} (V_i - I_i R)^2$$



② $R_i = \dfrac{V_i}{I_i}$

$$\hat{R}_{LS} = \frac{1}{N}\sum_{i=1}^{N} R_i$$

③ $G_i = \dfrac{I_i}{V_i}$

$$\left(\frac{1}{N}\sum_{i=1}^{N}\frac{I_i}{V_i}\right)^{-1} = \hat{R}_{LS}$$

p.17

① $V = IR$

$V - IR = 0$

$\boxed{\sum_{i=1}^{N} (V_i - I_i R)^2 = 0}$

①' $I = \frac{V}{R}$

$I - \frac{V}{R} = 0$

$\boxed{\sum_{i=1}^{N} \left(I_i - \frac{V_i}{R}\right)^2} = 0$

② $R = \frac{V}{I}$

$R - \frac{V}{I} = 0$

$\boxed{\sum_{i=1}^{N} \left(R - \frac{V_i}{I_i}\right)^2} = 0$

③ $\frac{1}{R} = \frac{I}{V}$

$\frac{1}{R} - \frac{I}{V} = 0$

$\boxed{\sum_{i=1}^{N} \left(\frac{1}{R} - \frac{I_i}{V_i}\right)^2} = 0$

Various Model functions

① $\begin{bmatrix} V_1 - I_1 R \\ V_2 - I_2 R \\ V_3 - I_3 R \\ \vdots \\ V_N - I_N R \end{bmatrix} = \underline{0}$

$\underline{h}(\underline{y}, \underline{\theta}) = \underline{e}$

$\hat{R}_{LS} = \arg\min_R \|e\|^2$

$\frac{1}{\hat{R}_{LS}} = \frac{1}{N} \sum \frac{I_i}{V_i}$

$\hat{R}_{LS} = \frac{1}{\frac{1}{N} \sum_{i=1}^{N} \frac{I_i}{V_i}}$

p.8

cf) linear observation model $\underline{Y} = H\underline{\theta} + \underline{N}$

a model function $\underline{y} - H\underline{\theta} = \underline{e}$

specialize

a linear model function $\underline{h}(\underline{y}) - H(\underline{y})\,\underline{\theta} = \underline{e}$

Lec. 39

Review

Detection & Estimation in AGN

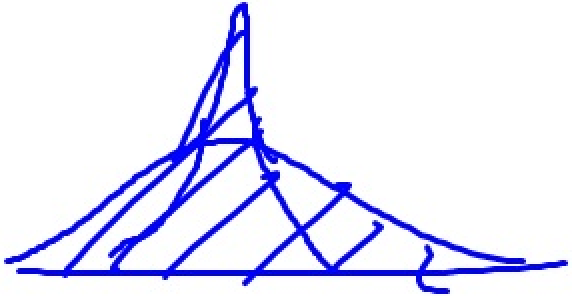
Least Squares Estimation   Model function = θ

Preview

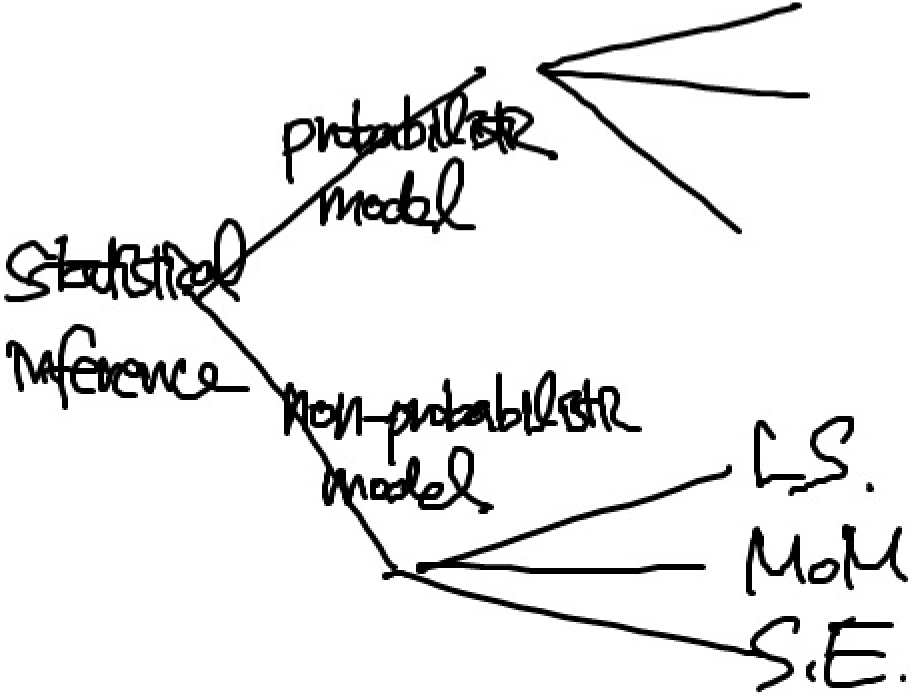
Method of Moments Estimation   SLLN a.s.

WLLN i.p.

⋮



arithmetic average → statistical average



model based

non-model based