CM11v250723w

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WSSUS (Wide-Sense Stationary Uncorrelated Scattering) channel model

1. DT Model First.

$c[t,\tau]$

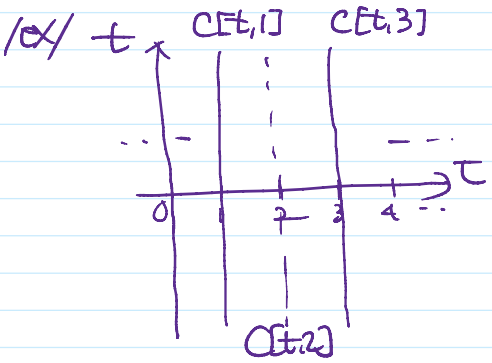
$y[t] = \sum_{\tau=-\infty}^{\infty} c[t,\tau]\, x[t-\tau]$

We keep t & τ but assume they are integers.

2. $c[t,\tau]$ is viewed as a (2-D) random field where a random variable $c[t,\tau]$ is given for each pair $[t,\tau]$.

3. What does "WSSUS" mean?

The DT CIR consists of countable infinite # of jointly distributed DT RPs $c[t,\tau]$ parameterized by τ.

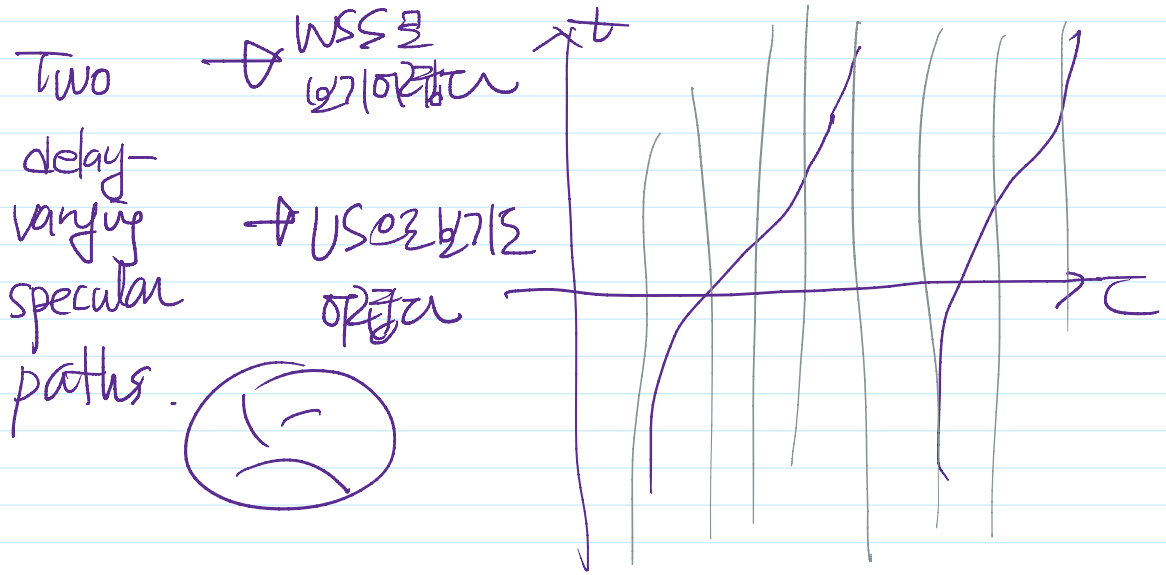


$c[t,1]$ & $c[t,3]$ are jointly distributed DT RPs with time parameter t & indexes 1 & 3, respectively.

① WSS means each DT RP $c[t,\tau_0]$ is a WSS DT RP.

② US " two DT RPs $c[t,\tau_0]$ & $c[t,\tau_1]$ are uncorrelated RPs.

} 현실에서 만족하기 어려운 가정!



4. Example

① WSSUS channel model.

Ⓖ: $\{c[t,\tau] : [t,\tau] \in \mathbb{Z}^2\}$ is a collection of countably infinite # of jointly Gaussian RVs.

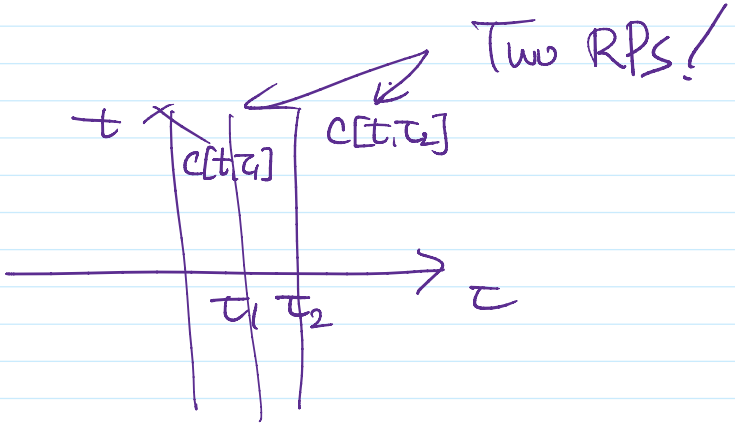
WSS: $c[t,\tau]$ is a WSS DT RP w/ time parameter t.

$$\begin{cases} E\{c[t,\tau_0]\} = \mu[\tau_0], \forall t \\ E\{c[t+s,\tau_0]\, c^*[t,\tau_0]\} = R_{cc}[s,\tau_0] \quad \text{a ft of time difference } s. \end{cases}$$

US : $c[t,\tau_0]$ & $c[t,\tau_1]$ are uncorrelated RPs for $\tau_0 \neq \tau_1$.

$$\mathrm{Cov}\{c[t,\tau_0], c^*[s,\tau_1]\} = 0, \quad \forall (t,s) \;\&\; \forall \tau_0 \neq \tau_1.$$

Correlation functions for a discrete-time discrete-delay LTV channel model



1. $c[t,\tau] \leftarrow$ CIR

$$\phi_c[\Delta t; \tau_1, \tau_2] \triangleq E\{c[t+\Delta t, \tau_1]\, c^*[t, \tau_2]\}$$

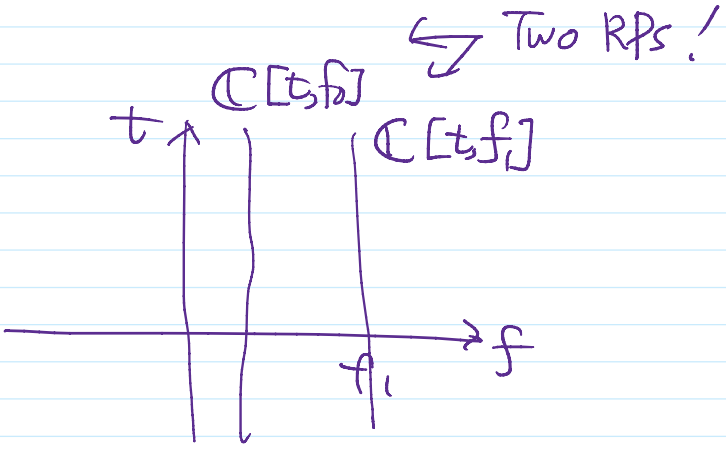
Cross-correlation function b/w $c[t,\tau_1]$ & $c[t,\tau_2]$

2. $C[t,f] \triangleq \mathcal{FT}_{\tau}\{c[t,\tau]\}$

↑ Channel frequency response

Q. Why?

A
$$y(t) = \int_{-\infty}^{\infty} c(t,\tau)\, x(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} c(t,\tau) \left\{ \int_{-\infty}^{\infty} X(f) e^{j2\pi f(t-\tau)} df \right\} d\tau$$
$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} c(t,\tau) e^{-j2\pi f\tau} d\tau \right] X(f) e^{j2\pi f t} df$$
$$= \int_{-\infty}^{\infty} C(t,f)\, X(f) e^{j2\pi f t} df$$



$$= \int_{-\infty}^{\infty} C(t;f)\, X(f) e^{j2\pi f\tau} df$$

Similar for DT.

$\therefore \phi_C[\Delta t; f_0, f_1] \triangleq E\{C[t+\Delta t, f_0]\, C^*[t, f_1]\}$

Xor btw $C[t, f_0]$ & $C[t, f_1]$

WSSUS의 정의

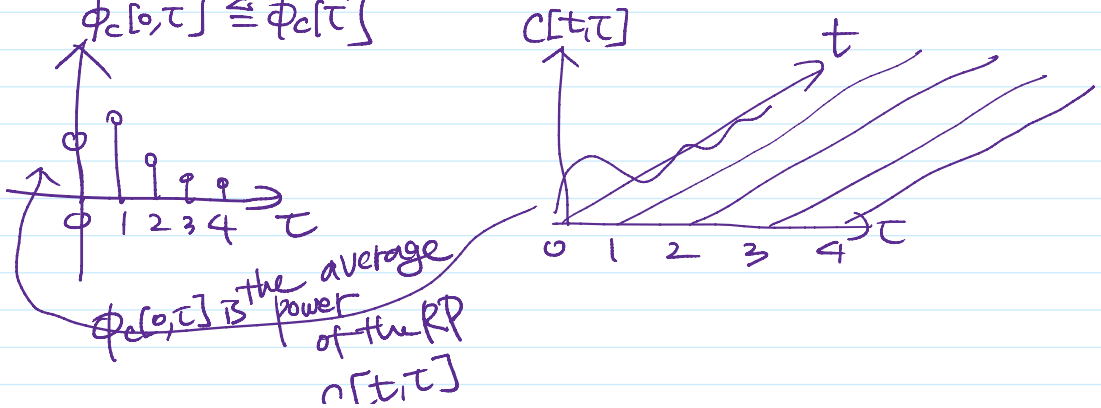
$$\phi_C[\Delta t; \tau_1, \tau_2] = E\{C[t+\Delta t, \tau_1]\, C^*[t, \tau_2]\} \triangleq \phi_C[\Delta t, \tau_1]\, \delta[\tau_1 - \tau_2]$$

$\tau_1 = \tau_2 \Rightarrow \phi_C[\Delta t, \tau_1]$ $\because$ WSS

$\tau_1 \neq \tau_2 \Rightarrow 0$. $\because$ US

$$\phi_C[0, \tau] = E\{|C[t, \tau]|^2\}$$

called the multipath intensity profile
or the power-delay profile
or the delay power spectrum of the channel.



~~$\phi_c[0,\tau]$ is power of the RV~~
$c[t,\tau]$

Using $\phi_c[\tau]$, we compute mean delay and rms delay spread.

2. WSSUS의 경우

$$\phi_C[\Delta t, f_1, f_2) = E\left\{\sum_{\tau_1} C[t+\Delta t, \tau_1] e^{j2\pi f_1 \tau_1} \sum_{\tau_2} C^*[t,\tau_2] e^{-j2\pi f_2 \tau_2}\right\}$$

$$= \sum_{\tau_1}\sum_{\tau_2} \phi_c[\Delta t, \tau_1]\delta[\tau_1-\tau_2]\, e^{-j2\pi(f_1\tau_1 - f_2\tau_2)} \quad \because \text{WSSUS channel!}$$

$$= \sum_{\tau_1} \phi_c[\Delta t, \tau_1]\, e^{j2\pi \underbrace{(f_1-f_2)}_{\Delta f}\tau_1}$$

$$\triangleq \phi_C[\Delta t, \Delta f) \quad \text{where } \Delta f = f_1 - f_2$$

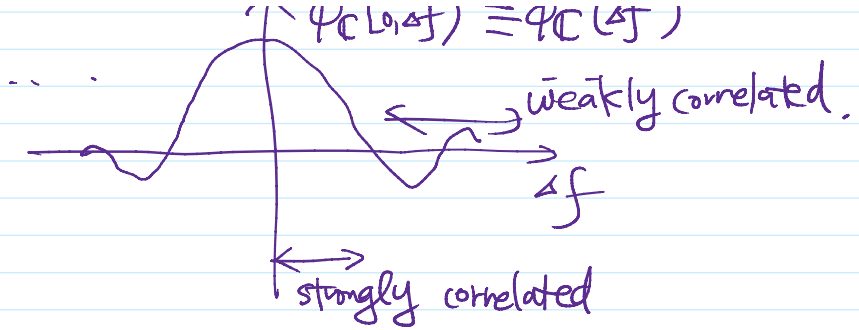
function only of frequency difference!

called the spaced-time spaced-frequency correlation function.

$$\phi_C[0,\Delta f) = \sum_{\tau_1} \phi_c[0,\tau_1] e^{j2\pi \Delta f \tau_1}, \text{ i.e, Fourier TF of } \phi_c[0,\tau]$$

$\phi_C[0,\Delta f) \triangleq \phi_C(\Delta f)$

weakly correlated



coherence bw can be obtained from $\phi_c(\Delta f)$

$\phi_c[\tau] \overset{\mathcal{FT}}{\Longleftrightarrow} \phi_c(\Delta f)$

delay 퍼짐 $\Rightarrow$ coherence BW 좁다!

frequency-selective
vs
-nonselective