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2025년 7월 23일 수요일 22:29

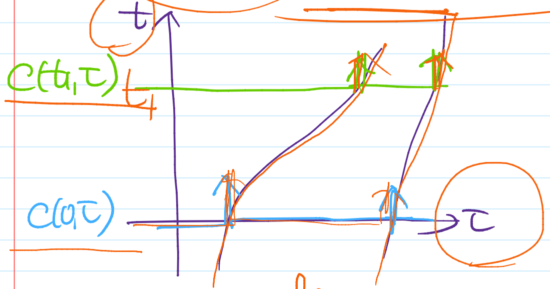
□ WSSUS (wide-sense stationary uncorrelated scattering) channel model Part 2.

□ Review w/ CT model

1. LTV CIR

$h(\tau,t)$

$$y(t)=\int_{-\infty}^{\infty} c(t,\tau)\,x(t-\tau)\,d\tau$$



cf) LTI

$$y(t)=\int_{-\infty}^{\infty} c(\tau)\,x(t-\tau)\,d\tau = c(t)*x(t)$$

⇒ $c(t,\tau)$ is interpreted as CIR at t

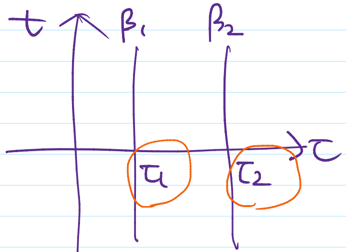
$\cancel{h}(\tau)=\beta_1\delta(\tau-\tau_1)+\beta_2\delta(\tau-\tau_2)$ (c)

ex) (S) LTI CIR w/ 2 paths   No t!

$c(t,\tau)=\beta_1\delta(\tau-\tau_1)+\beta_2(\tau-\tau_2)$

⇒ $y(t)=\beta_1 x(t-\tau_1)+\beta_2 x(t-\tau_2)$

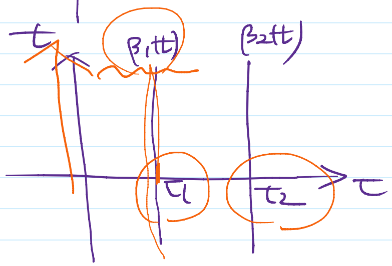
constants



(S)' Specular LTV CIR w/ fixed delay

$c(t,\tau)=\beta_1(t)\delta(\tau-\tau_1)+\beta_2(t)\delta(\tau-\tau_2)$

⇒ $y(t)=\beta_1(t)\,x(t-\tau_1)+\beta_2(t)\,x(t-\tau_2)$



$\Rightarrow y(t) = \beta_1(t)\, x(t-\tau_1) + \beta_2(t)\, x(t-\tau_2)$

Now time-varying multiplicative process

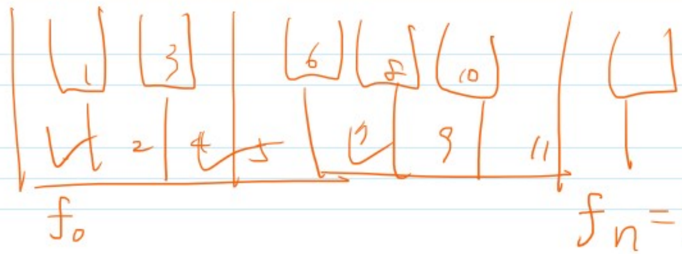
((S)) specular LTV CIR w/ fixed gain delay & Doppler

$$c(t,\tau) = (\beta_1 e^{j2\pi\nu_1 t})\,\delta(\tau-\tau_1) + (\beta_2 e^{j2\pi\nu_2 t})\,\delta(\tau-\tau_2)$$

mag phase   delay

$\beta_1(t)\, e^{j2\pi\nu_1 t}$ Doppler $\tau_1(t)$ ..

ν nu μ mu



$f_n = f_0\, 2^{\frac{n}{12}}$

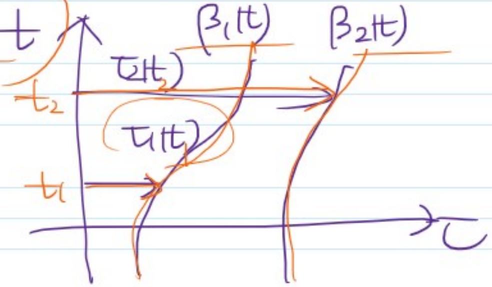
$((S))^2$ specular LTV CIR w/ TV delay

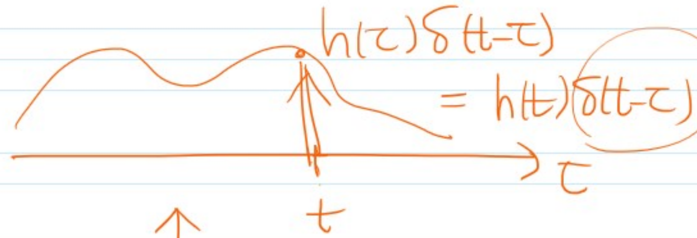
$$c(t,\tau) = \beta_1(t)\,\delta(\tau-\tau_1(t)) + \beta_2(t)\,\delta(\tau-\tau_2(t))$$

$\Rightarrow y(t) = \beta_1(t)\, x(t-\tau_1(t)) + \beta_2(t)\, x(t-\tau_2(t))$

time-varying delay

complex gain multiplied to delayed version of $x(t)$





$((S))^n$ specular LTV CIR w/ L paths

$$c(t,\tau) = \sum_{l=1}^{L} \beta_l(t)\,\delta(\tau-\tau_l(t))$$

mag, phase   delay

$h(t) = \int^{\infty} (h(\tau)\delta(t-\tau))\,d\tau$

$c(t,\tau) = \sum_{\ell=1} \beta_\ell(t)\, \delta(\tau-\tau_\ell(t))$

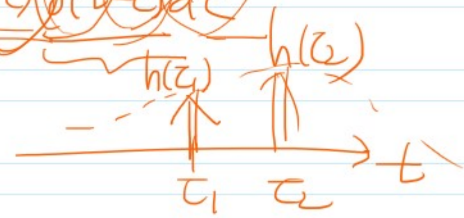
mag. phase   delay

$\Rightarrow\ y(t) = \sum_{\ell=1}^{L} \beta_\ell(t)\, x(t-\tau_\ell(t))$

$h(t) = \int_{-\infty}^{\infty} h(\tau)\delta(t-\tau)\,d\tau$

LHS

t



((S)) $c(t,\tau) = \tilde{c}(t,\tau')\,\delta(\tau-\tau')$

$\Rightarrow\ y(t) = \tilde{c}(t,\tau')\, x(t-\tau')$

((G)) $c(t,\tau) = \int_{-\infty}^{\infty} c(t,\tau')\,\delta(\tau-\tau')\,d\tau'$ Sifting property

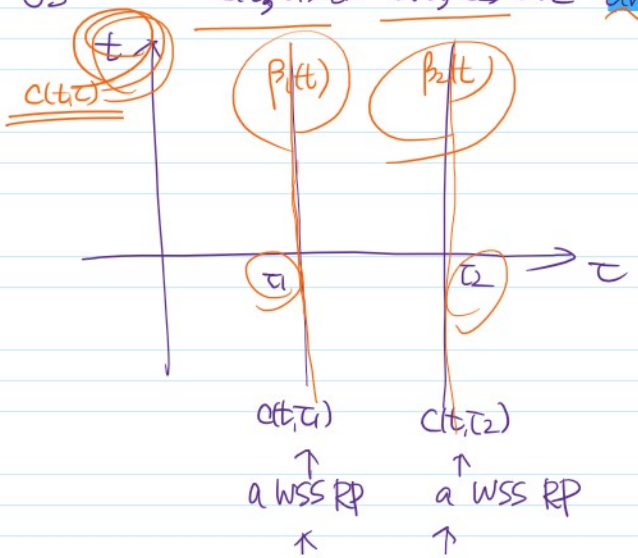
LHS τ multiplicative process

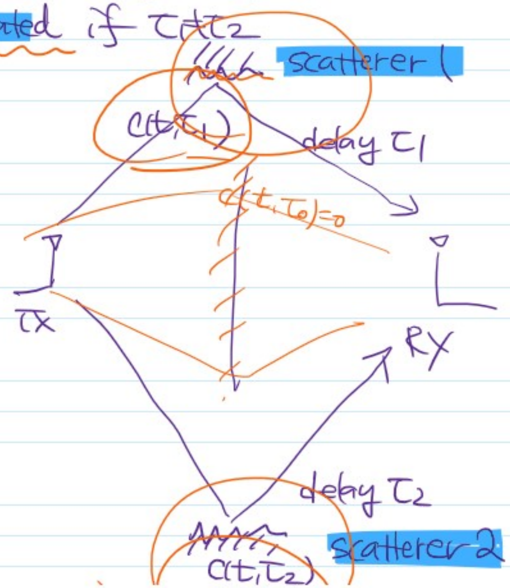
$\Rightarrow\ y(t) = \int_{-\infty}^{\infty} c(t,\tau')\, x(t-\tau')\,d\tau'$

2. $c(t,\tau)$ is viewed as a scalar(-valued) 2-D random field.

WSS $c(t,\tau_0)$ as a RP is WSS. Assume zero-mean.

US $c(t,\tau_1)$ & $c(t,\tau_2)$ are uncorrelated if $\tau_1 \neq \tau_2$





a WSS RP a WSS RP

uncorrelated RPs

delay τ_2

scatterer 2

$c(t,\tau_2)$

$c(t,\tau)$ of the CIR $c(t,\tau)$

3. Correlation function & WSSUS assumption.

spaced time.

$$\phi_c(t_1,t_2;\tau_1,\tau_2) = E\{c(t_1,\tau_1)\,c^*(t_2,\tau_2)\} = \phi_c(t_1-t_2;\tau_1)\,\delta(\tau_1-\tau_2)$$

Corrections!

Not spaced delay!

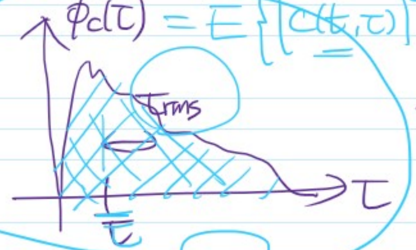
Absolute delay matters.

$\phi_c(\Delta t;\tau_1)$ is the autocorrelation function of the WSS RP $c(t,\tau_1)$

$\beta_1(t)$

$\phi_c(\tau_1) \triangleq \phi_c(0,\tau_1)$ is the average power of the WSS RP $c(t,\tau_1)$

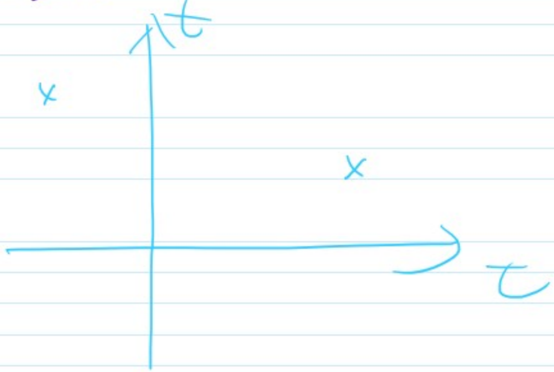
$\phi_c(\tau) = E\{|c(t,\tau)|^2\} \geq 0$



the MIP

P-D profile

delay power spectrum



$$\bar{\tau} \triangleq \frac{\int_{-\infty}^{\infty} \tau\,\phi_c(\tau)\,d\tau}{\int_{-\infty}^{\infty} \phi_c(\tau)\,d\tau}, \quad \text{mean excess delay}$$

$$\tau_{rms} \triangleq \sqrt{\frac{\int_{-\infty}^{\infty} (\tau-\bar{\tau})^2\,\phi_c(\tau)\,d\tau}{\int_{-\infty}^{\infty} \phi_c(\tau')\,d\tau'}}, \quad \text{rms delay spread.}$$

standard dev.에 해당.

4. Channel Freq. Response

Recall that for LTI, $y(t) = \int_{-\infty}^{\infty} c(\tau)\,x(t-\tau)\,d\tau$

4. Channel Freq. Response

Recall that for LTI,

$$\boxed{y(t)} = \int_{-\infty}^{\infty} \boxed{c(\tau)}\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} c(\tau) \left(\boxed{\int_{-\infty}^{\infty} X(f) e^{j2\pi f(t-\tau)} df} \right) d\tau$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} c(\tau) e^{-j2\pi f\tau} d\tau \right) X(f) e^{j2\pi f t} df$$

$$= \int_{-\infty}^{\infty} \boxed{C(f) \qquad X(f)} e^{j2\pi f t} df$$

$$\Rightarrow \quad Y(f) = \boxed{C(f)}\, X(f)$$

called the CFR.

system transfer function

H(s).

Similarly, for LTV,

$$y(t) = \int_{-\infty}^{\infty} c(t,\tau)\, x(t-\tau)\, d\tau$$

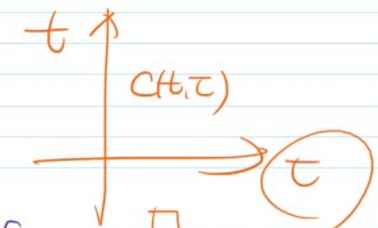
$$\vdots$$

$$= \int_{-\infty}^{\infty} \boxed{\left(\int_{-\infty}^{\infty} c(t,\tau) e^{-j2\pi f\tau} d\tau \right)} X(f) e^{j2\pi f t} df$$

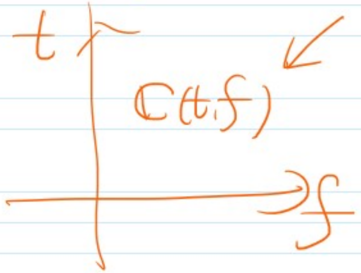
$$= \int_{-\infty}^{\infty} C(t,f) \qquad X(f) e^{j2\pi f t} df$$

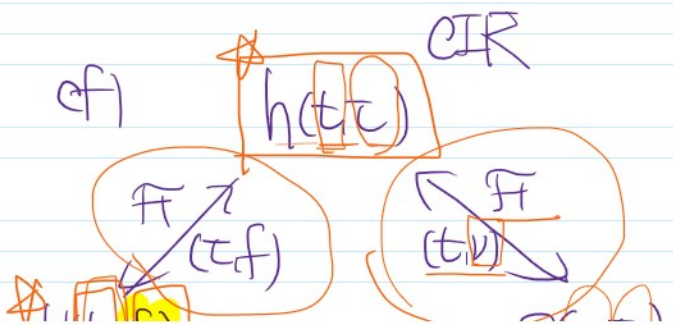
called the time-varying CFR,

a.k.a. time-variant transfer function.



If $c(t,\tau)$ is a random field,
then $C(t,f)$ is also a random field.





T — f

CFR

time-variant transfer ft.

(τ,f) (t,ν)

H(t,f) S(ν,τ) delay-Doppler ft

F (t,ν) Ft (τ,f)

B(ν,f)

Doppler-variant transfer ft

...eless channels are time-variant, with an impulse response $h(t,\tau)$ that changes ... to distinguish between the absolute time t and the delay τ. Thus, the theory of ... Variant (LTV) system must be used. This is not just a trivial extension of the ... gives rise to considerable theoretical challenges, and causes the breakdown of ... concepts. Fortunately, most wireless channels can be classified as *slowly time-var...* known as *quasi-static*. In that case, many of the concepts of LTI systems can ... only minor modifications.

...terization of deterministic linear time-variant systems

Bello

...e response of a time-variant system, $h(t,\tau)$, depends on two variables, τ and t ... Fourier transformations with respect to either (or both) of them. This results ... but equivalent, representations. In this section, we will investigate these rep... ...ir advantages, and drawbacks.

...tem-theoretic point of view, it is most straightforward to write the relationship ... input (transmit signal) $x(t)$ and the system output (received signal) $y(t)$ ...

$$y(t) = \int_{-\infty}^{\infty} x(\tau) K(t,\tau) d\tau$$

...the *kernel* of the integral equation, which can be related to the impulse response ... the well-known relationship $K(t,\tau) = h(t-\tau)$ holds. Generally, we define ... impulse response:

$$h(t,\tau) = K(t, t-\tau)$$

h(τ,t) ...

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(t,\tau)\, d\tau$$

τ

...erform a Fourier transformation with respect to the variable t, τ, or both ... interpretation is possible if the impulse response changes only slowly with time ... duration of the impulse response (and the signal) should be much shorter than ... which the channel changes significantly. Then we can consider the behavior of the ... time t like that of an LTI system. The variable t can thus be viewed as "absolute ... which impulse response $h(\tau)$ is currently valid. Such a system is also called ...

delay τ

...orming the impulse response with respect to the variable τ results in the time-variant ... function $H(t,f)$:

$$H(t,f) = \int_{-\infty}^{\infty} h(t,\tau) \exp(-j2\pi f\tau)\, d\tau \quad (6.1...)$$

Molisch

A Fourier transformation with respect to t results in a different representation – namely, the *delay Doppler function*, better known as *spreading function* $S(\nu,\tau)$:

$$S(\nu,\tau) = \int_{-\infty}^{\infty} h(t,\tau) \exp(-j2\pi\nu t)\, dt \quad (6.13)$$

S(τ,ν)

This function describes the spreading of the input signal in the delay and Doppler domains.

Finally, the function $S(\nu,\tau)$ can be transformed with respect to the variable τ, resulting in the *Doppler-variant transfer function* $B(\nu,f)$:

~~for~~ freq-Doppler ft

$$B(\nu,f) = \int_{-\infty}^{\infty} S(\nu,\tau) \exp(-j2\pi f\tau)\, d\tau \quad (6.14)$$

A summary of the interrelations between the system functions is given in Fig. 6.5. Figure 6.6 shows an example of a measured impulse response; Figure 6.7 shows the spreading function computed from it.

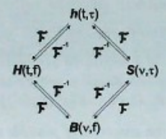
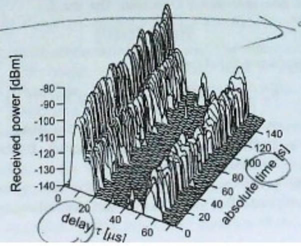


Figure 6.5 Interrelation between deterministic system functions.

quasi-static



delay τ

…orming the impulse response with respect to the variable τ results in the time-variant transfer function $H(t,f)$:

$$H(t,f)=\int_{-\infty}^{\infty} h(t,\tau)\exp(-j2\pi f\tau)\,d\tau \qquad (6.\ldots)$$

…t relationship is given by:

$$y(t)=\int_{-\infty}^{\infty} X(f)H(t,f)\exp(j2\pi ft)\,df \qquad (6.\ldots)$$

…tion is straightforward for the case of the quasi-static system – the spectrum of … is multiplied by the spectrum of the "currently valid" transfer function, to give … the output signal. If, however, the channel is quickly time-varying, the … purely mathematical relationship. The spectrum of the output signal is given … integral

$$Y(\tilde f)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(f)H(t,f)\exp(j2\pi ft)\exp(-j2\pi\tilde f t)\,df\,dt \qquad (6.\ldots)$$

…educe to $Y(f)=H(f)X(f)$ [Matz and Hlawatsch 1998].

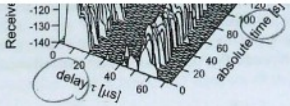


Figure 6.6 Squared magnitude of the impulse response $|h(t,\tau)|^2$ measured in hilly terrain near Darmstadt, Germany. Measurement duration 140 s; center frequency 900 MHz. τ denotes the excess delay.

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6.3.2 *Stochastic system functions*

We now return to the stochastic description of wireless channels. Interpreting them as time-variant stochastic systems, a complete description requires the multidimensional pdf of the impulse response – i.e., the joint pdf of the complex amplitudes at all possible values of delay and time. However, this is usually much too complicated in practice. Instead, we restrict our attention to a second-order description – namely, the *Auto Correlation Function* (ACF).

6. Correlation function of the TV CFR $C(t,f)$ & WSSUS assumption.

General

$$\phi_C(t_1,t_2,f_1,f_2)=E\left[C(t_1,f_1)\,C^*(t_2,f_2)\right]$$

$$=E\left[\left(\int_{-\infty}^{\infty} c(t_1,\tau_1)e^{-j2\pi f_1\tau_1}d\tau_1\right)\left(\int_{-\infty}^{\infty} c(t_2,\tau_2)e^{-j2\pi f_2\tau_2}d\tau_2\right)^*\right]$$

special WSSUS $\rightarrow$

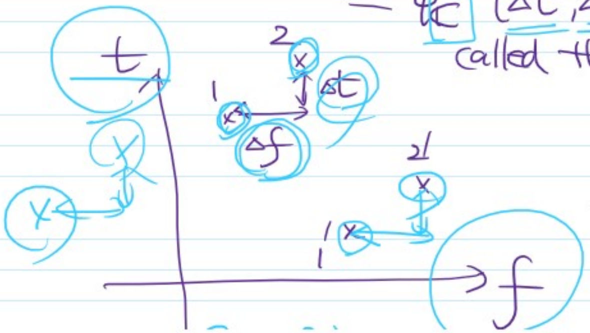
$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\boxed{\phi_c(t_1-t_2,\tau_1)\,\delta(\tau_1-\tau_2)}\,e^{-j2\pi(f_1\tau_1-f_2\tau_2)}d\tau_1 d\tau_2$$

$$=\int_{-\infty}^{\infty}\phi_c(t_1-t_2,\tau_1)\,e^{-j2\pi(f_1-f_2)\tau_1}d\tau_1$$

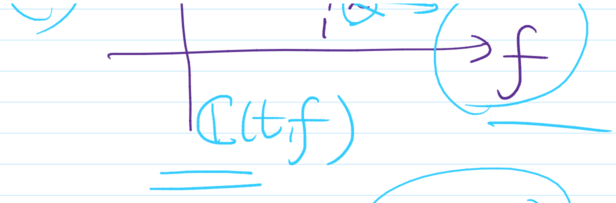
$$=\phi_C(t_1-t_2,f_1-f_2) \qquad \Delta t=t_1-t_2,\ \Delta f=f_1-f_2$$

$$=\phi_C(\Delta t,\Delta f)$$

called the spaced-time spaced-freq correlation function.



$\Rightarrow$ The correlation b/w $C(t_1,f_1)$ & $C(t_2,f_2)$ is the same as that b/w $C(t_1',f_1')$ & $C(t_2',f_2')$



is the same as that b/w $C(t_1',f_1')$ & $C(t_2',f_2')$

if $t_1-t_2 = t_1'-t_2'$ &

$f_1-f_2 = f_1'-f_2'$.

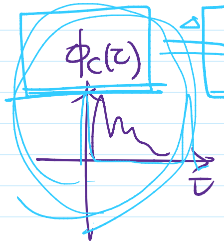
7. Relation b/w $\phi_c(\Delta t,\tau)$ & $\phi_C(\Delta t,\Delta f)$

$\delta(\tau_1-\tau_2)$

$$\boxed{\phi_C(\Delta t,\Delta f) = \int_{-\infty}^{\infty} \phi_c(\Delta t,\tau)\, e^{-j2\pi\Delta f\tau}\, d\tau,}$$ i.e.,

$$\phi_c(\Delta t,\tau) \xrightarrow[\tau,\Delta f]{FT} \phi_C(\Delta t,\Delta f)$$ spaced freq. corr. ft.

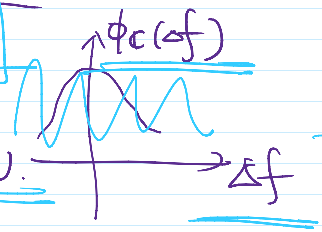
$$\boxed{\phi_c(\tau)} = \boxed{\phi_c(0,\tau) \xrightarrow[\tau,\Delta f]{FT} \phi_C(0,\Delta f)}$$

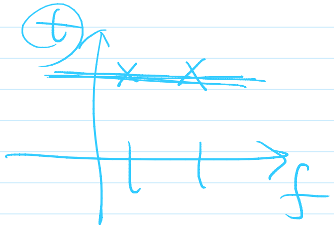


MIP

delay spread

coherence BW.



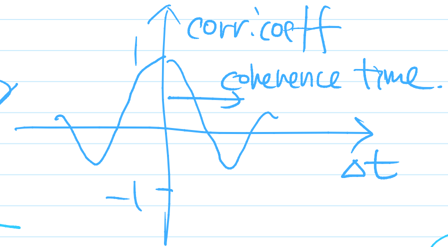


□ ((S)) Frequency-Flat time-varying channel & Notion of Coherence Time.

$c(t,\tau) = \beta_0(t)\,\delta(\tau-\tau_0)$

$$\Rightarrow \boxed{y(t) = \beta_0(t)\, x(t-\tau_0)}$$

multiplicative RP



If $\beta_0(t)$ & $x(t)$ are ~~uncorrelated~~ independent RPs

then $R_{YY}(t+\Delta t,t) = E[\beta_0(t+\Delta t)\, x(t+\Delta t-\tau_0)\, \beta_0^*(t)\, x^*(t-\tau_0)]$

$= R_{\beta_0\beta_0}(\Delta t)\, R_{xx}(\Delta t)$

FT ↑

X, Y ⇒ $E[XY] = E[X]E[Y]$

Z, W

$E[XYZW]$

$= E[\ldots]E[\ldots]$

$$= R_{\beta_0\beta_0}(\Delta t)\, R_{xx}(\Delta t)$$

$$\tilde{\mathcal{F}} \updownarrow \quad (\Delta t, \nu)$$

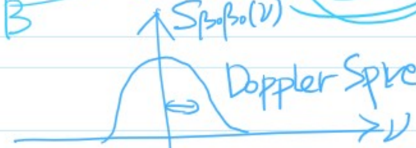
$$S_{\tilde{y}\tilde{y}}(\nu) = \underbrace{S_{\beta_0\beta_0}(\nu)}_{W} * \underbrace{S_{xx}(\nu)}_{B}$$

Doppler Spread

$E[XYZW]$

$= E[XY]E[ZW]$

$\beta_0(t)$



((S)) $\beta_0(t) = e^{j2\pi\nu_0 t}$

$R_{\beta_0\beta_0}(\Delta t) = e^{j2\pi\nu_0\Delta t}$

$S_{\beta_0\beta_0}(\nu) = \delta(\nu-\nu_0)$

$\Rightarrow S_{\tilde{y}\tilde{y}}(\nu) = S_{xx}(\nu-\nu_0)$ Doppler Freq. Shift.

□ $c(t,\tau)$ as the multiplicative RP for delayed signal $x(t-\tau)$

$$\phi_c(\Delta t, \Delta f) = \int_{-\infty}^{\infty} \phi_c(\Delta t, \tau)\, e^{-j2\pi\Delta f\tau}\, d\tau$$

$$\overset{\mathcal{F}}{\underset{(\tau,\Delta f)}{\Longleftrightarrow}} \phi_c(\Delta t, \tau)$$

← autocorrelation function of the RP $c(t,\tau)$

$$\mathcal{F}\ (\Delta t, \nu) \swarrow$$

$$\boxed{S(\nu,\tau)}$$

← PSD of the RP $c(t,\tau)$

a.k.a. the scattering function

$\sum_{l=1}^{L} x_l(t) X(t-\tau_l)$

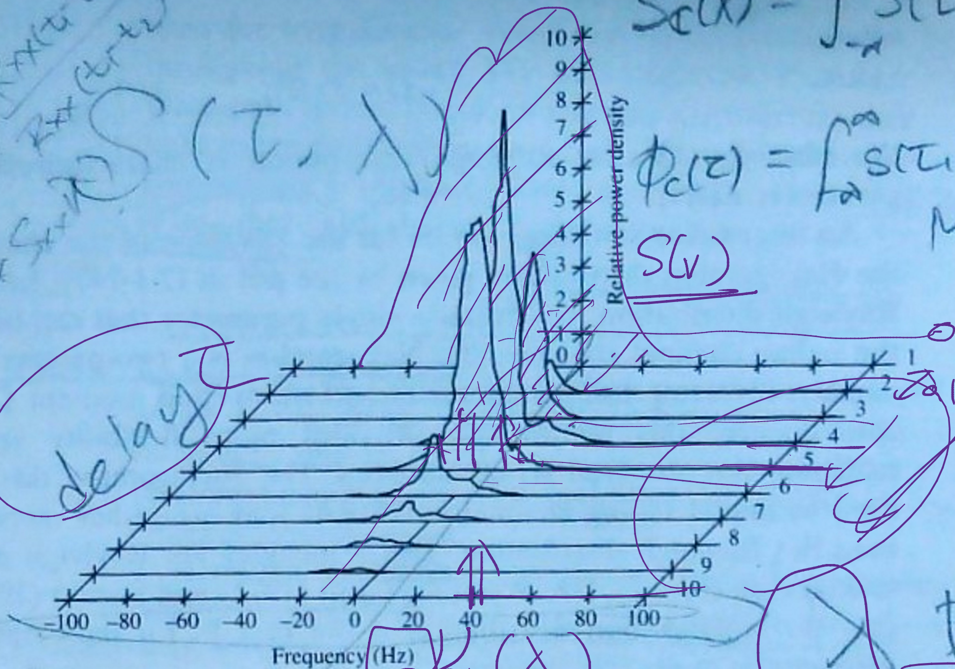
$\sum_{l=1}^{L} R(t_1-t_2)$ $R_{xx}(t_1-\tau_l, t_2-\tau_l)$ $R_{xx}(t_1-t_2)$

$\sum_{l=1}^{L} S_l(T) * S_{xx}(f)$

$S(\tau, \lambda)$

$S_c(\lambda) = \int_{-\infty}^{\infty} S(\tau, \lambda)\, d\tau$ Doppler PSD

$\phi_c(\tau) = \int_{-\infty}^{\infty} S(\tau, \lambda)\, d\lambda$ MIP



$2 \times 0.1 \mu sec$. 30m / 10 MHz BW.

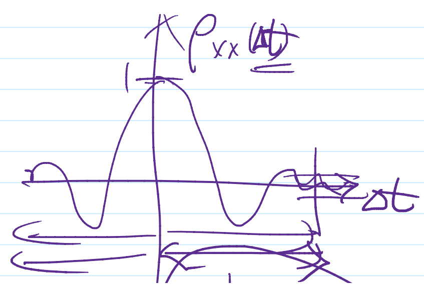
FIGURE 14-1-6 Scattering function of a medium-range tropospheric scatter channel. The taps delay increment is 0.1 μs.

Q. $S(\nu,\tau)$: each τ has different Doppler Spread.

$\phi_c(\Delta t, \tau)$ " " " " " coherence time.

What is the average Doppler Spread &

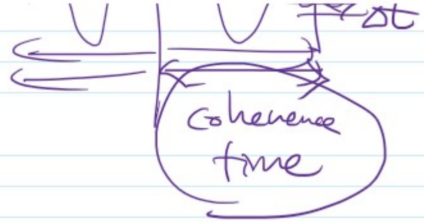
" coherence time?



What is the average Doppler Spread &

" Coherence time?

How can we obtain them?



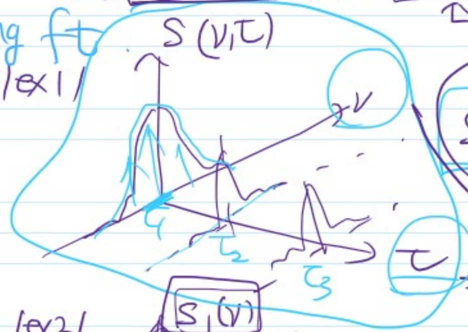
A. Consider $\int_{-\infty}^{\infty} \boxed{S(\nu,\tau)} d\tau \triangleq \boxed{S(\nu)}$ (average)

$\triangle$ Doppler Power Spectrum

Scattering ft

cf) delay-Doppler ft

/ex1/



$$\boxed{S(\nu,\tau)} = S_1(\nu)\delta(\tau-\tau_1) + S_2(\nu)\delta(\tau-\tau_2) + S_3(\nu)\delta(\tau-\tau_3)$$

$$\Rightarrow S(\nu) = \sum_{i=1} S_i(\nu).$$

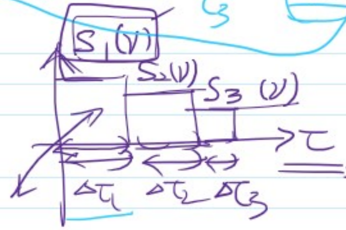
$c(t,\tau)$

$= \beta_1(t)\delta(\tau-\tau_1)$

$+ \beta_2(t)\delta(\tau-\tau_2)$

$+ \beta_3(t)\delta(\tau-\tau_3)$

/ex2/



$$\Rightarrow S(\nu) = S_1(\nu)\Delta\tau_1 + S_2(\nu)\Delta\tau_2 + S_3(\nu)\Delta\tau_3$$

$\mathcal{F}_{(\nu)}$

Also consider

$$\int_{-\infty}^{\infty} \phi_c(\Delta t,\tau)\, d\tau = \boxed{\phi_c(\Delta t)}$$

$\uparrow$ average autocorrelation function of the multiplicative RPs.

□ Q. What are the relations among the corr. fts?

Recall $x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \Rightarrow \boxed{x(0) = \int_{-\infty}^{\infty} X(f)\, df}$

Recall $x(t)=\int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df \Rightarrow x(0)=\int_{-\infty}^{\infty} X(f)\, df$

& $X(f)=\int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt \Rightarrow X(0)=\int_{-\infty}^{\infty} x(t)\, dt$

for a single-variable ft & its FT.

$$= \phi_C(\Delta t,\Delta f)\Big|_{\Delta f=0}$$

$$=\int_{-\infty}^{\infty} \phi_C(\Delta t,\tau)\, e^{-j2\pi \Delta f \tau}\, d\tau\Big|_{\Delta f=0} = \phi_C(\Delta t, 0)$$

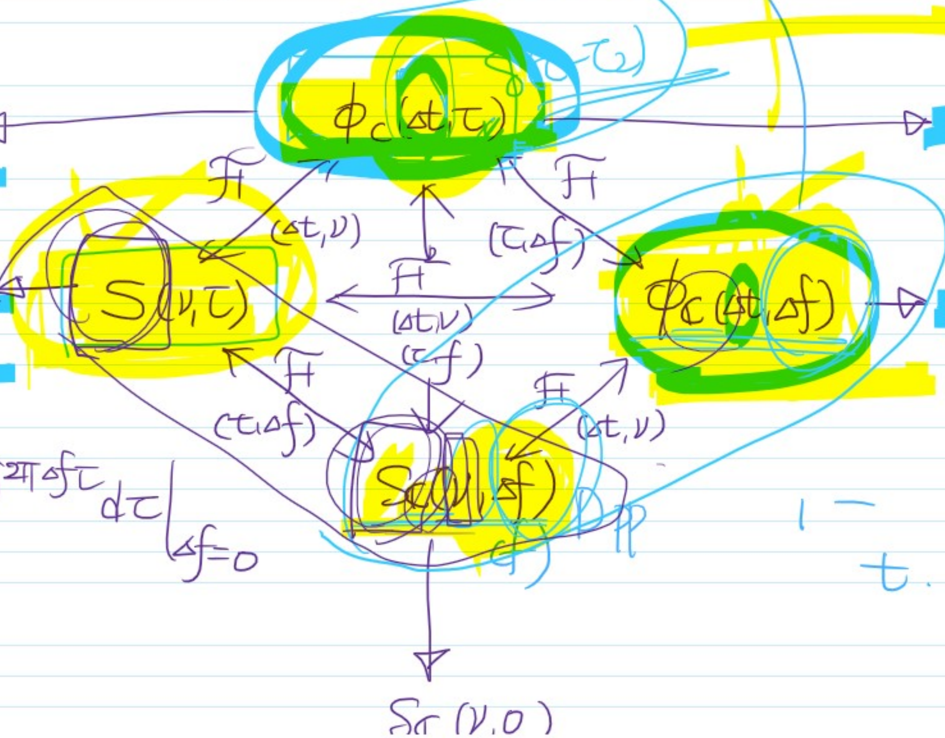
coherence time

$\phi_C(\Delta t)=\int_{-\infty}^{\infty}\phi_C(\Delta t,\tau)\, d\tau$

FT $(\Delta t,\nu)$

$S(\nu)=\int_{-\infty}^{\infty} S(\nu,\tau)\, d\tau$

Doppler spread



delay spread

$\phi_C(\tau)\triangleq\phi_C(0,\tau)$

FT $(\tau,\Delta f)$

$\phi_C(\Delta f)\triangleq\phi_C(0,\Delta f)$

coherence BW

$$=\int_{-\infty}^{\infty} S(\nu,\tau)\, e^{-j2\pi\Delta f\tau}\, d\tau\Big|_{\Delta f=0}$$

$$= S_C(\nu,0)$$

$$\triangleq S_C(\nu)$$

$\triangleq S_c(\nu)$

$S_c(\nu, 0)$

□ Mean Excess Delay & rms delay spread from the scattering ft

$[X, Y] \quad f_{X,Y}(x,y)$

$$\bar{\tau} = \frac{\iint \tau\, \boxed{S(\nu,\tau)}\, d\nu\, d\tau}{\iint S(\nu,\tau)\, d\nu\, d\tau}$$

$\phi_c(\tau)$

As if 2-D PDF!

$$\bar{X} = \int_{-\infty}^{\infty} x f_X(x)\, dx = \iint_{-\infty}^{\infty} x f_{X,Y}(x,y)\, dx\, dy$$

$$\tau_{rms} = \sqrt{\frac{\iint (\tau-\bar{\tau})^2 S(\nu,\tau)\, d\nu\, d\tau}{\iint S(\nu,\tau)\, d\nu\, d\tau}}$$

□ Mean Doppler Freq Shift & rms Doppler spread from the scattering ft

$$\bar{\nu} = \frac{\iint \nu\, S(\nu,\tau)\, d\nu\, d\tau}{\iint S(\nu,\tau)\, d\nu\, d\tau}$$

$$\int S(\nu,\tau)\, d\tau = S(\nu)$$

$$\nu_{rms} = \sqrt{\frac{\iint (\nu-\bar{\nu})^2 S(\nu,\tau)\, d\nu\, d\tau}{\iint S(\nu,\tau)\, d\nu\, d\tau}}$$