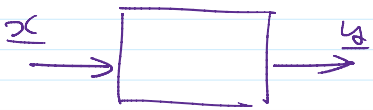
CM15v250801fri

2025년 8월 1일 금요일 오후 4:49

① linear system w/ input $x \in \mathbb{C}^N$, output $y \in \mathbb{C}^M$

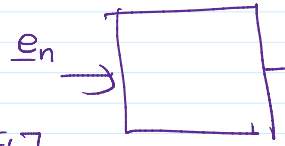


Thm. Given a linear system $(\mathcal{Q}, M, \mathcal{R})$ with $\mathcal{Q} = \mathbb{C}^M$, $\mathcal{R} = \mathbb{C}^N$ there exists a matrix $H \in \mathbb{C}^{M \times N}$ s.t. $y = Hx$

Proof. $y = M(x) = M\left(\sum_{n=1}^{N} x_n e_n\right)$ where e_n is the nth standard basis vector of $\mathbb{C}^N$

$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, $\cdots$ $e_N = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$

$= \sum_{n=1}^{N} x_n M(e_n)$



$= \sum_{n=1}^{N} h_n x_n$

$= \underbrace{\begin{bmatrix} h_1 & h_2 & \cdots & h_N \end{bmatrix}}_{\triangleq H \in \mathbb{C}^{M \times N}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = Hx.$

↑ ((S))

② linear system w/ input $x(t)$ a length-N vector-valued CT signal, output $y(t)$ " " M " " " " "

↓ ((S))

" " " " $x[l]$ " " " " DT signal

" $y[l]$ " " " " DT signal.

((S)) $M = N = 1$

$y[m] = \sum_{n=-\infty}^{\infty} h[m,n]\, x[n]$

$h[m,n] \triangleq h[m, m-n]$

$y[m] = \sum_{n=-\infty}^{\infty} h[m,n]\,x[n]$ $\quad h[m,n] \triangleq h[m, m-n]$

$= \sum_{n=-\infty}^{\infty} h[m, m-n]\,x[m-n]$

Thm. ---- , then there exists $H[k,l] \in \mathbb{C}^{M\times N}$, $k,l \in \mathbb{Z}$ such that

$$\underline{y}[m] = \sum_{l=-\infty}^{\infty} H[m,l]\,\underline{x}[l].$$

proof.

$$\underline{y}[m] = \mathcal{M}(\underline{x}[m]) = \mathcal{M}\left(\begin{bmatrix} x_1[m] \\ x_2[m] \\ \vdots \\ x_N[m] \end{bmatrix}\right) = \mathcal{M}\left(\sum_{n=1}^{N} x_n[m]\,\underline{e}_n\right) = \mathcal{M}\left(\sum_{n=1}^{N}\sum_{l=-\infty}^{\infty} x_n[l]\,\delta[m-l]\,\underline{e}_n\right)$$

($x_n[m] = \sum_{l=-\infty}^{\infty} x_n[l]\,\delta[m-l]$)

$\underline{x}[m] \rightarrow \boxed{\quad} \rightarrow \underline{y}[m]$

$$= \sum_{n=1}^{N}\sum_{l=-\infty}^{\infty} x_n[l]\,\mathcal{M}(\underline{e}_n \delta[m-l]) =$$

($\underline{e}_n$: time l.)

$$= \sum_{l=-\infty}^{\infty}\sum_{n=1}^{N} \underline{h}_n[m,l]\,x_n[l]$$

$$= \sum_{l=-\infty}^{\infty} \begin{bmatrix} \underline{h}_1[m,l] & \cdots & \underline{h}_N[m,l] \end{bmatrix} \begin{bmatrix} x_1[l] \\ \vdots \\ x_N[l] \end{bmatrix}$$

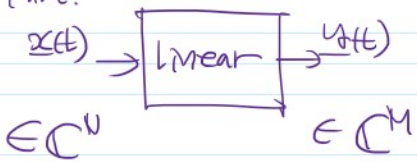
$$= \sum_{l=-\infty}^{\infty} H[m,l]\,\underline{x}[l]$$

((S)) LTI. $\quad H[m,0] = H[m]$

$$\underbrace{\underline{y}[m]}_{\mathbb{C}^{M\times 1}} = \sum_{l=-\infty}^{\infty} \underbrace{H[m-l]}_{\mathbb{C}^{M\times N}}\,\underbrace{\underline{x}[l]}_{\mathbb{C}^{N\times 1}} = H[m] * \underline{x}[m].$$

□ Thm. (H(t))

□ Thm.



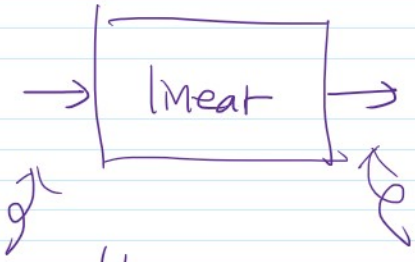
Thm. There exists a matrix-valued function $H(t,\tau)$ of (t,τ) s.t.

$$\underline{y}(t) = \int_{-\infty}^{\infty} H(t,\tau)\,\underline{x}(\tau)\,d\tau$$

Corollary. If LTI, then

$$\underline{y}(t) = \int_{-\infty}^{\infty} H(t-\tau)\,\underline{x}(\tau)\,d\tau = H(t) * \underline{x}(t)$$

□ Summary



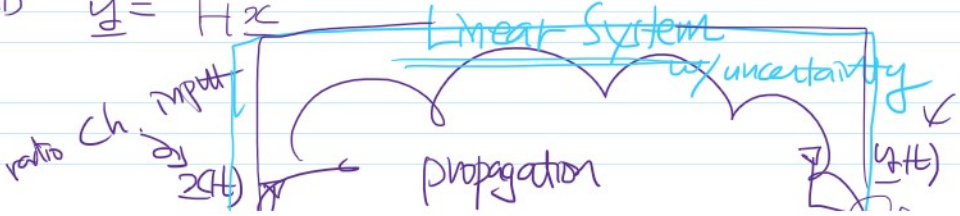
□ Channel. ~~$H(t,\tau) \triangleq K(t,t-\tau)$~~ ~~$\equiv H(t,t-\tau)$~~

(i) $\underline{y}(t) = \int_{-\infty}^{\infty} H(t,\tau)\,\underline{x}(t-\tau)\,d\tau$ $\xrightarrow{\text{LTI}}$ $\underline{y}(t) = \int_{-\infty}^{\infty} H(\tau)\,\underline{x}(t-\tau)\,d\tau = H(t) * \underline{x}(t)$ ↖H

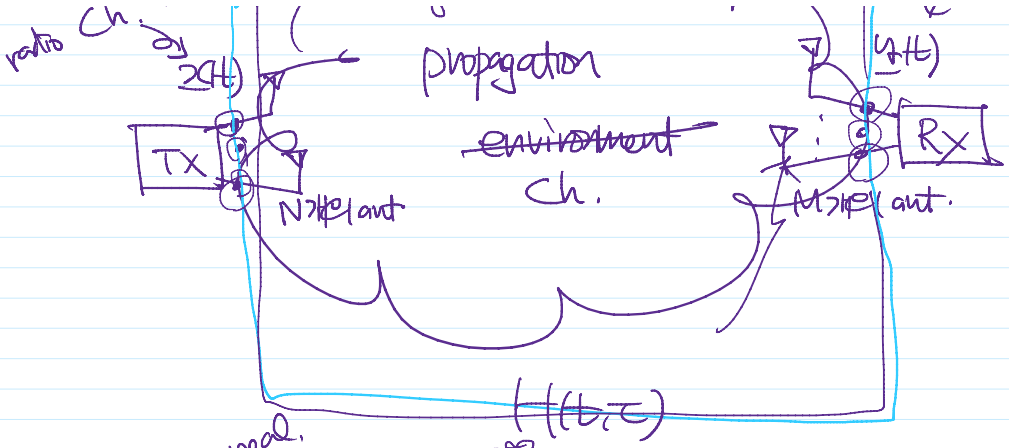
(ii) $\underline{y}[m] = \sum_{l=-\infty}^{\infty} H[m,l]\,\underline{x}[m-l]$ $\xrightarrow{\text{LTI}}$ $\underline{y}[m] = \sum_{l=-\infty}^{\infty} H[l]\,\underline{x}[m-l] = H[m] * \underline{x}[m]$ ↙H

(iii) $\underline{y} = H\underline{x}$

□



MIMO system/channel.



MIMO system/channel.

$\underline{y}(t) = \mathcal{H}(\underline{x}(t))$

real. BP

$\underline{y}(t) = \int_{-\infty}^{\infty} \underline{H}(t,\tau)\,\underline{x}(\tau)\,d\tau + \underline{n}(t)$

$\underline{y}_c(t) = \int_{-\infty}^{\infty} \underline{H}_c(t,\tau)\,\underline{x}_c(\tau)\,d\tau + \underline{n}_c(t)$

cf) $\underline{y}_c = \underline{H}_c \underline{x}_c + \underline{n}_c$

$\underline{y} = \underline{H}\underline{x} + \underline{n}$

MIMO Ch.

Great Inventions!

1960 USSR Soviet Union

economy!

TRFZ

Scarce resource abundant

W : E

1960 decade 1969

VLSI

Cellular Concept

AT&T/Bell Labs

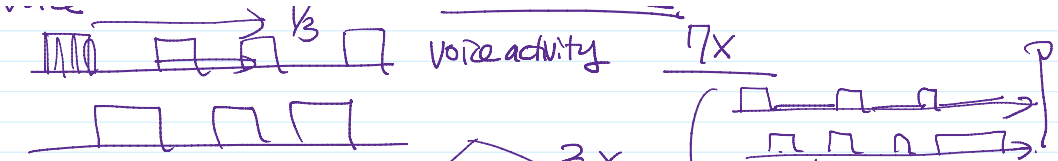
cell pattern

CDMA 100×

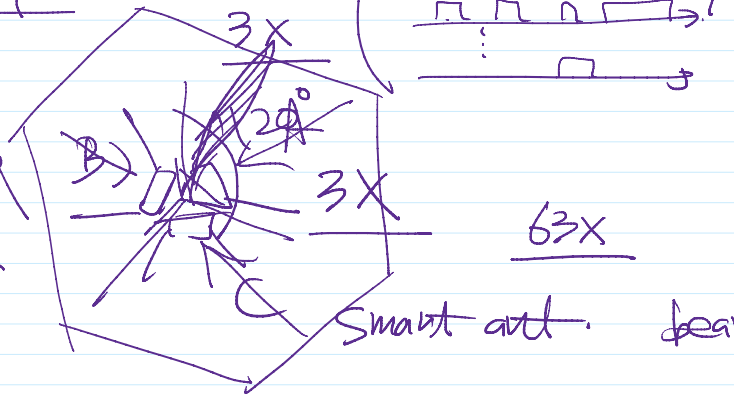
+power control → DS/SS

Voice → 1/3 voice activity

FRF=1 7×



- Sectorized antenna

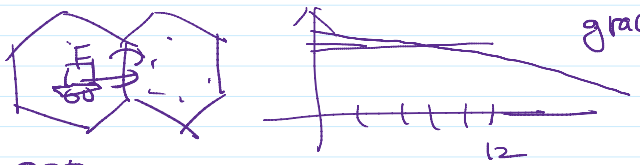


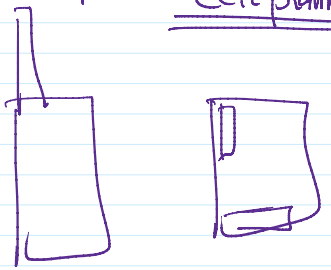
63X

- Advanced Vocoder & ECC. 2X

Smart ant. beam forming.

-



- CSP cell planning

- MIMO