CM01v250901mon

2025년 9월 1일 월요일 오후 2:09

Wide-sense stationary RP & Its Spectral Characteristic

□ Def. for CT RP:

$X(t)$ is wide-sense stationary if

① $E\{X(t)\} = \mu_X, \ \forall t$

(ii) $E\{X(t+\tau)X^*(t)\} = R_{XX}(\tau), \quad \forall (t,\tau) \in \mathbb{R}^2$

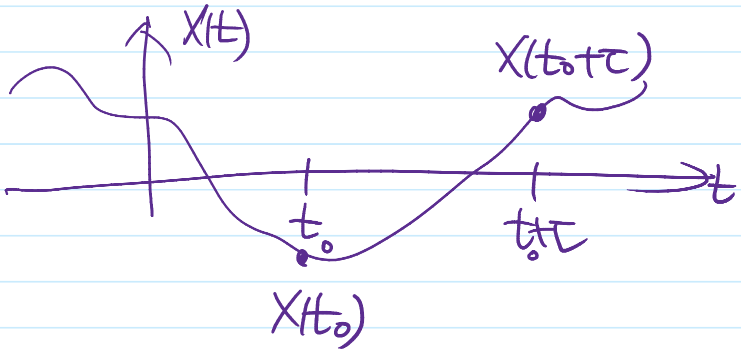
there exists a single-variable function ..

□ Motivated from 1st-order & 2nd-order stationary RPs

Def. $X(t)$ is 1st-order stationary if

$F_{X(t)}(x) = F_{X(t+\tau)}(x), \ \forall (t,\tau), \ \forall x$

$$F_{X(t_0)}(x) = F_{X(t_0+\tau)}(x), \quad \forall (t_0, \tau), \ \forall x \in \mathbb{R}$$



Lemma. If $X(t)$ is 1st order stationary, then $E\{X(t_0)\} = E\{X(t_0+\tau)\} = \mu_X, \ \forall t_0, (\tau)$

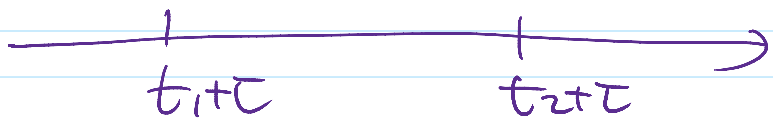
$$E\{X(t_0)\} = \int_{-\infty}^{\infty} x \, dF_{X(t_0)}(x) = \int_{-\infty}^{\infty} x f_{X(t_0)}(x)\, dx = \mu_X,$$

Caution. Converse not true.

Def. $X(t)$ is 2nd-order stationary RP if

$$F_{X(t_1),X(t_2)}(x_1, x_2) = F_{X(t_1+\tau),X(t_2+\tau)}(x_1, x_2),$$

$$\forall (t_1, t_2, \tau)$$
$$\forall (x_1, x_2)$$

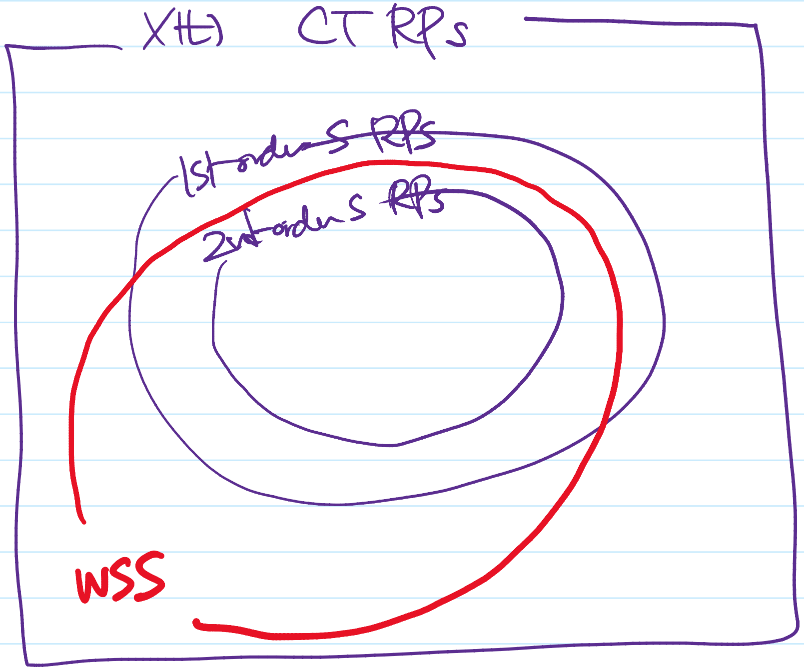


Lemma. (i) 2nd-order stationarity $\Rightarrow$ 1st-order stationarity

(ii) " $\Rightarrow$ $E\{X(t)\} = \mu_x \;\forall t$.

(iii) " $\Rightarrow$ $E\{X(t+\tau)\, X^*(t)\} = R_{XX}(\tau), \;\forall (t,\tau)$

Caution: Converse of (ii)&(iii) is not true.



* for complex-valued RPs.

☐ LTI filtering of a WSS RP.

$$\underset{\text{WSS}}{X(t)} \rightarrow \boxed{h(t)} \rightarrow Y(t)$$

Lemma. $Y(t)$ is also WSS.

Def. $X(t)$ & $Y(t)$ are jointly WSS if

(i) $X(t)$ is WSS

(ii) $Y(t)$ " "

(iii) $E\{X(t+\tau)\,Y(t)\} = R_{XY}(\tau), \ \forall (t,\tau)$

Lemma. ~~$X(t)$~~ Input & output RPs are jointly WSS if Input is WSS.

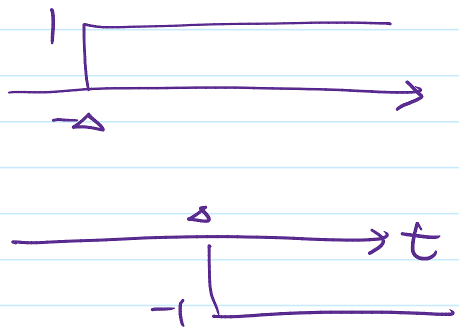
☐ PSD. xPSD

Def. The PSD of $X(t)$ is given by

$$S_{XX}(f) \triangleq \lim_{\Delta\to\infty} \frac{E\{|X_\Delta(f)|^2\}}{2\Delta}$$

ESD

$X_\Delta(f)$ is the CTFT of $X(t)\underbrace{\{u(t+\Delta)-u(t-\Delta)\}}$



Thm. (W.K).

$$S_{XX}(f) \triangleq \mathcal{F}_\tau\left[A_t\{\underbrace{E\{X(t+\tau)X(t)\}}\}\right]$$

Cor. If $X(t)$ is WSS,

then $S_{XX}(f) = \mathcal{F}_\tau\{R_{XX}(\tau)\}$

Def.

$$S_{XY}(f) \triangleq \lim_{\Delta\to\infty} \frac{E\{X_\Delta(f)Y_\Delta^*(f)\}}{2\Delta}$$

x PSD

Thm (W.K) $S_{XY}(f) \triangleq \mathcal{F}_\tau\left[A_t\{E\{X(t+\tau)Y^*(t)\}\}\right]$

Cor. If $X(t)$ and $Y(t)$ are jointly WSS

Cor. If $X(t)$ and $Y(t)$ are jointly WSS,

$$S_{XY}(f) = \mathcal{F}_\tau\{R_{XY}(\tau)\}$$

□

$$X(t) \longrightarrow \boxed{h(t)} \longrightarrow Y(t)$$

WSS WSS

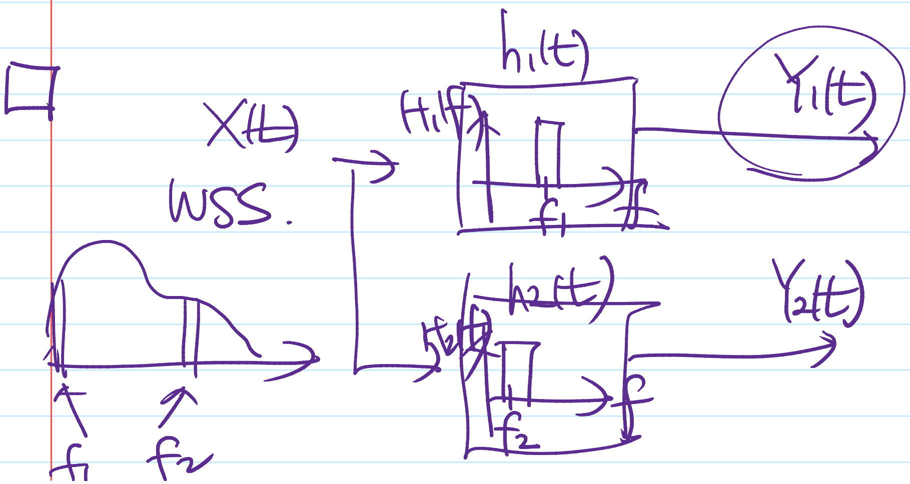
$S_{XX}(f)$

$$S_{YY}(f) = |H(f)|^2 \, S_{XX}(f)$$

$$S_{XY}(f) = S_{XX}(f)\, H^*(f)$$

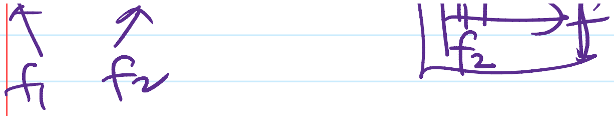
$$S_{YX}(f) = H(f)\, S_{XX}(f)$$

□



$\cancel{E\{Y_1(t_1)\,Y_2\}}$

0



Q1. $Y_1(t)$ & $Y_2(t)$ are jointly WSS?

A1. Yes, because $E\{Y_1(t+\tau)Y_2^*(t)\} = h_1(\tau) * R_{XX}(\tau) * h_2^*(-\tau)$, $\forall t$.

$$\Updownarrow \mathcal{F}$$

Q2. xPSD? $\quad S_{Y_1Y_2}(f) = H_1(f)\, S_{XX}(f)\, H_2^*(f)$

Q3. What if $H_1(f)H_2^*(f) = 0, \forall f$

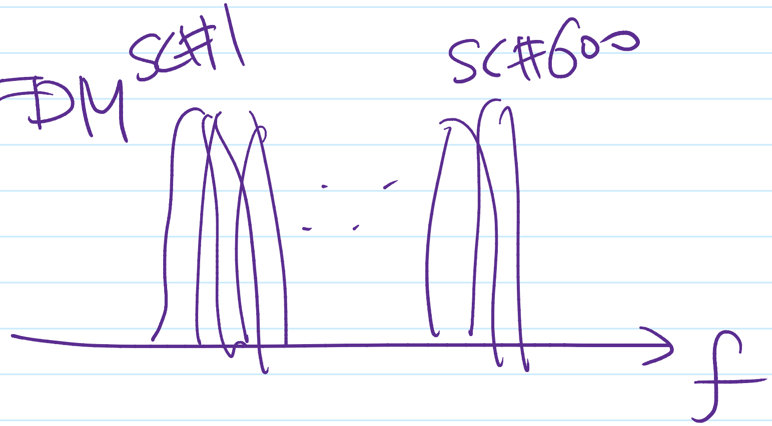
$$\Rightarrow S_{Y_1Y_2}(f) = 0, \forall f.$$

Cor. $X(t)$ is zero WSS

$Y_1(t)$ " " "

$Y_2(t)$ " " " $\Big\}$ jointly WSS $\quad \underline{E\{Y_1(t+\tau)Y_2^*(t)\} = 0, \forall t, \tau}$

☐ OFDM



(DFT-S)-OFDM w/ FDSS SE