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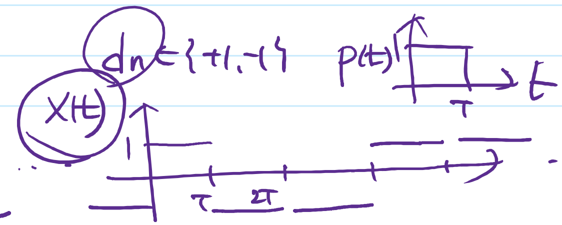


$$\sum_{n=-\infty}^{\infty} d_n\, p_n(t)$$

BPSK mod. signal

$\mathrm{Re}\{x(t)\, e^{j(2\pi f_c t + \theta)}\}$

QAM, PAM

$$x(t) = \sum_{n=-\infty}^{\infty} d_n\, p(t - nT)$$

input to the ch.

$\sum b[k] s(t-kT)$

even with such a simple scenario.[1] Later in [9], Kumar and Roy further investigated this problem for symmetrically cross-coupled N-user channels, and again showed that a set of coupled, nonlinear equations needs to be solved to obtain jointly optimum transmitter and receiver waveforms. A comprehensive treatment on this topic can be found in [22].

In this correspondence, we consider a joint transmitter and receiver optimization for an SISO system with an additive WSCS noise in contrast to the previous works with additive WSS noises. Note that cochannel interference, crosstalk, adjacent-cell interference, and many other types of data-like interference of digital communication systems are best modeled as WSCS noises rather than WSS noises. It is assumed that the period of the cyclostationarity of the noise is the same as the inverse of the symbol transmission rate and that the noise has a positive-definite autocorrelation function. The data sequence is modeled as a proper-complex wide-sense stationary colored random process and the channel is modeled as a linear time-invariant system with a general frequency-selective impulse response. After a weighted water-filling is performed to find the optimum transmission energy allotment, the optimum transmitter and receiver waveforms that jointly minimize the MSE at the output of a linear receiver are obtained in terms of the vectorized Fourier transforms of waveforms and the matrix-valued power spectral density of the cyclostationary noise.

The solution of this variational problem is applicable in designing the optimum transmit filter when a new system employing a linear MMSE receiver is overlaid on an existing system having a linear modulation with the same symbol transmission rate. It is also applicable in canceling data-like jamming signals. It is also worth noting that this joint optimization is the locally optimum action taken by each pair of a transmitter and a receiver in a noncooperative game, where each pair competes with other pairs sharing the same frequency band to achieve a better quality of service in a wireless *ad hoc* network. Note that in all these cases, the interference is accurately modeled as a WSCS noise with the aforementioned characteristic.

The rest of this correspondence is organized as follows. The signal and system models are described in Section II. The frequency-domain expression of the MSE is derived in Section III, and the jointly optimum

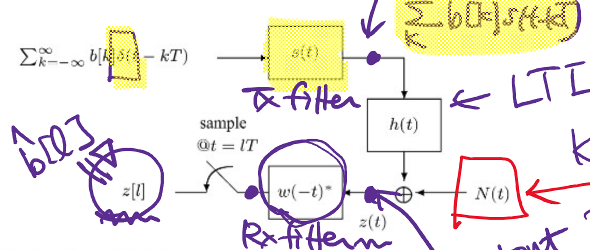


Fig. 1. System block diagram.

impulse response $h(t)$ having bandwidth B (Hz). Without loss of generality, we assume that the carrier frequency used for the complex baseband representation is the center frequency of the channel. The output of the channel is corrupted by a zero-mean proper-complex[2] noise $N(t)$. Hence, the received signal can be written as

$$z(t) = \sum_{k=-\infty}^{\infty} b[k]p(t - kT) + N(t) \tag{2}$$

where $p(t) \triangleq h(t) \otimes s(t)$ is the response of the channel to the input $s(t)$ with $\otimes$ denoting convolution.

It is assumed that the noise process is band-limited with the same bandwidth B as the transmitted signal, positive definite, and having the period of cyclostationarity T.[3] Related definitions are as follows.

Definition 1: [11, p. 7] A second-order proper-complex random process $N(t)$ is band-limited with bandwidth B if it is unaffected by passing it through an ideal low-pass filter with cutoff frequency B.

Definition 2: A second-order proper-complex random process $N(t)$ with bandwidth B and the autocorrelation function

$$r_N(t, s) \triangleq \mathbb{E}\{N(t)N(s)^*\}$$

is called positive definite if

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t)^* r_N(t,s)x(s)\,dt\,ds > 0 \tag{3}$$

$\underline{w}^{*}(-t)$

$\underline{w}^{H}(-t)$

$\underline{w}(-t)^{H}$

$*$

$w(t) * z(t) =$

$\otimes$ \otimes

$\mathbb{E}\{\underline{N}\,\underline{N}^H\}$

$\underline{z}^H R \underline{z} > 0$

Tx pulse/wave fm $s(t)$

with the aforementioned characteristic.

The rest of this correspondence is organized as follows. The signal and system models are described in Section II. The frequency-domain expression of the MSE is derived in Section III, and the jointly optimum transmitter and receiver waveforms are derived in Section IV. Numerical results are provided in Section V. Finally, concluding remarks are offered in Section VI.

II. Signal and System Model

The system block diagram in complex baseband is shown in Fig. 1, where and in what follows the superscript * denotes complex conjugation. Modification of this signal and system models to the real-valued baseband case is straightforward. The transmitter employs a linear modulation scheme with a transmitter waveform $s(t)$ and a symbol transmission rate $1/T$ (symbols/s). The data sequence $(b[k])_k$ is modeled as a discrete-time WSS random process with the auto-correlation function

$$m[k] \triangleq \mathsf{E}\{b[k+l]b[l]^*\}$$

and the power spectral density

$$M(f) \triangleq \sum_{k=-\infty}^{\infty} m[k]e^{-j2\pi fk}. \quad (1)$$

The transmitted signal is passed through a strictly band-limited channel, which is modeled as a linear time-invariant system with the

[1]This is because there exists no adequate mathematical tools for this type of a complicated nonlinear functional optimization problem [7].

$$r_N(t,s) \triangleq \mathsf{E}\{N(t)N(s)^*\}$$

is called positive definite if

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(t)^* r_N(t,s) w(s)\,dt\,ds > 0 \quad (3)$$

for all nonzero $w(t) \in \mathbb{L}^2(B)$.[4]

Definition 3: A second-order random process $N(t)$ is called WSCS with mean zero and period T if

$$\mu_N(t) \triangleq \mathsf{E}\{N(t)\} = 0, \quad \forall t$$

and

$$r_N(t,s) = r_N(t+kT, s+kT), \quad \text{for all } t,s \in \mathbb{R} \text{ and } k \in \mathbb{Z}.$$

The simplest example is the case where an additive white Gaussian noise (AWGN) corrupts the channel output. Since the out-of-band signal is irrelevant [12], only the in-band portion of the noise can be regarded as $N(t)$. This band-limited white Gaussian noise is certainly positive definite because $\int_{-\infty}^{\infty} |w(t)|^2 dt > 0$ for all nonzero $w(t) \in \mathbb{L}^2(B)$ by definition. From this example, we see that the solution in [1] is a special case of what is considered in this correspondence.

[2]Roughly speaking, a WSS complex-valued random process $N(t)$ satisfying $\mathsf{E}\{N(t)N(s)\} = 0, \forall t, s$, is called proper. For details about proper-complex random processes, see [10].

[3]For the cases where the frequency band of $h(t)$ being apparently contained in the frequency band of $N(t)$, we just need to redefine the frequency band of $h(t)$ to fully include all relevant out-of-band signals that have nonzero spectral correlation with the in-band signals.

[4]$\mathbb{L}^2(B)$ is the set of all finite energy, band-limited signals with bandwidth B.

$z^H R z > 0$
$\forall z \neq 0$

$\mu_N(t) = 0, \forall t$

Tx pulse/waveform $s(t) \leftarrow$

Rx pulse

E

$(s(t), w(t))$

const.
$A[E\{|x(t)|^2\}] = AT$

□ $R \in \mathbb{C}^{N\times N}$

Assume $R = R^H$ ← the Hermitian transposition.

Def. R is positive definite if $\forall z \in \mathbb{C}^{N\times 1} \neq 0$, $z^H R z > 0$.

Def. $\underline{N}$ a complex RVector. $R \triangleq E\{\underline{N}\,\underline{N}^H\}$ is positive definite if $z^H R z > 0$, $\forall z \neq 0$

semi-def.

Def. $N(t)$ 〃 $E\{N(t_1)N^*(t_2)\}$ 〃 〃 〃

if $\int^{\infty}\int^{\infty} w^*(t_1)\, E\{N(t_1)N^*(t_2)\}\, w(t_2)\, dt_1\, dt_2 > 0 \ \forall$

semi-def.

$$\text{if} \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w^*(t_1)\, E\{N(t_1)N^*(t_2)\}\, w(t_2)\, dt_1\, dt_2 > 0, \ \forall$$

$w(t) \neq$ zero signal.

□ Def. A real-valued 2nd-order (with cycle period $T>0$) RP $X(t)$ is wide-sense cyclostationary if

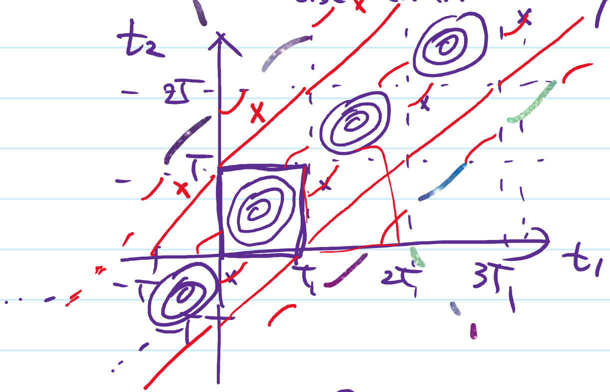
(i) $E\{X(t)\} = \boxed{\mu_X(t) = \mu_X(t+T)}$, $\forall t$

$\mu_X(t) = \mu_X(t+kT)$, $\forall t \in \mathbb{R}$, $\forall k \in \mathbb{Z}$.

(ii) $E\{X(t_1)X(t_2)\} = E\{X(t_1+kT)X(t_2+kT)\}$, $\forall (t_1,t_2) \in \mathbb{R}^2$, $k \in \mathbb{Z}$

((ii)' $E\{(X(t_1)-\mu_X(t_1))(X(t_2)-\mu_X(t_2))\} = E \ \ldots$)

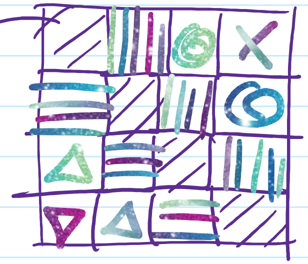
wide-sense, covariance cyclostationary ...



(t_1, t_2)

(t_1+T, t_2+T)

3x3



$E\{X(t_1)X(t_2)\}$
$= E\{X(t_1+T)X(t_2+T)\}$

□ A proper-complex RV.

☐ A proper-complex RV.

$Z = X + jY$

└ two jointly distributed Real RV.

$\cancel{f_{X,Y}}$ $F_{X,Y}(x,y)$, $f_{X,Y}(x,y)$

$E\{Z\} \triangleq E\{X\} + jE\{Y\}$

$Var\{Z\} \triangleq E\{|Z - E\{Z\}|^2\}$

$\widetilde{Var}\{Z\} \triangleq E\{(Z - E\{Z\})^2\}$

↑ complementary variance
pseudo-varian

Def. If $\widetilde{Var}\{Z\} = 0$, then Z is a proper-complex RV.

If $\widetilde{Var}\{Z\} \neq 0$, then Z is improper.

$E\{Z(t)\} = \mu_Z(t)$.

$E\{(Z(t_1) - E\{Z(t_1)\})(Z(t_2) - E\{Z(t_2)\})^*\}$

$E\{(Z(t_1) - E\{Z(t_1)\})(Z(t_2) - E\{Z(t_2)\})\}$

Def. If $E\{(Z(t_1) - E\{Z(t_1)\})(Z(t_2) - E\{Z(t_2)\})\} = 0, \forall (t_1, t_2)$

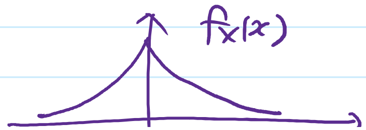
then $Z(t)$ is proper

☐ $\boxed{\text{A 2nd-order RP}}$

(i) $E\{|X(t)|^2\} < \infty, \forall t$

(ii)

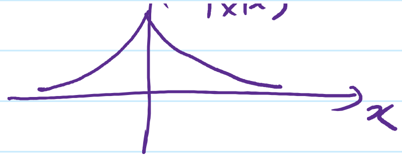
cf) X $E\{X\} = \int_{-\infty}^{\infty} x \, dF_X(x)$ ← Not guaranteed to exist.



cf) X $\quad E\{X\} = \int_{-\infty}^{\infty} x \, dF_X(x)$ ← Not guaranteed to exist.

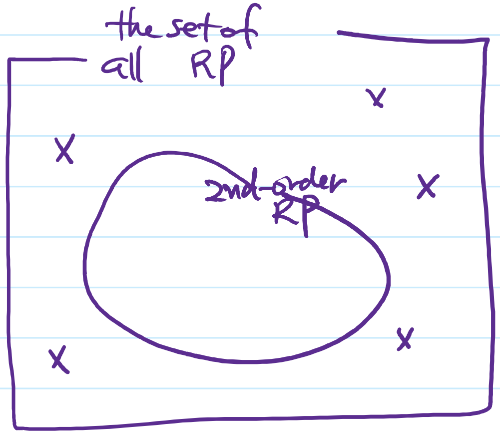
$\overline{\underline{E\{X^2\}}} = \int_{-\infty}^{\infty} x^2 \, dF_X(x)$ ← " " " "

$\mathrm{Var}\{X\} = \int_{-\infty}^{\infty} (x - E\{X\})^2 \, dF_X(x)$



$\int_{-\infty}^{\infty} x \, f_X(x) \, dx$

Levy-stable



The receiver consists of a linear time-invariant filter with the impulse response $w(-t)^*$ and a sampler with the sampling rate $1/T$ (samples/s). The decision statistic $z[l]$ for the detection of the lth symbol $b[l]$ is defined as

$$z[l] \triangleq w(-t)^* \otimes z(t)\Big|_{t=lT} = \int_{-\infty}^{\infty} w(t-lT)^* z(t)\,dt$$
$$= \sum_{k=-\infty}^{\infty} b[k]q((l-k)T) + n[l] \tag{4}$$

where the waveform $q(t)$ is defined as

$$q(t) \triangleq w(-t)^* \otimes p(t) = \int_{-\infty}^{\infty} w(\tau)^* p(t+\tau)\,d\tau \tag{5}$$

and the noise component $n[l]$ is defined as

$$n[l] \triangleq \int_{-\infty}^{\infty} w(t-lT)^* N(t)\,dt. \tag{6}$$

wH R w.

The noise variance can be written as

$$\mathsf{E}\{|n[0]|^2\} = \mathsf{E}\left\{\left(\int_{-\infty}^{\infty} w(t)^* N(t)\,dt\right)\left(\int_{-\infty}^{\infty} w(s)^* N(s)\,ds\right)^*\right\}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(t)^* r_N(t,s) w(s)\,dt\,ds. \tag{7}$$

Hence, the condition (3) leads to nonzero noise in the observation after filtering.

The performance metric used in this correspondence is $\boxed{\text{the MSE}}$ defined as

$$\varepsilon \triangleq \mathsf{E}\{|b[k] - z[k]|^2\} \tag{8}$$
$$= \mathsf{E}\{|b[k]|^2 - 2\Re(b[k]^* z[k]) + |z[k]|^2\}$$
$$= \mathsf{E}\{|b[k]|^2\} \tag{9a}$$
$$- 2\Re(\mathsf{E}\{b[k]^* z[k]\}) \tag{9b}$$
$$+ \mathsf{E}\{|z[k] - n[k]|^2\} \tag{9c}$$
$$+ \mathsf{E}\{|n[k]|^2\} \tag{9d}$$

$E\{z[k]\}$

$E\{b[k]\}$

Re{·}

where $\Re$ denotes the real part, (9a) is the reference component, (9b) is the cross-correlation component between the reference signal and the output, (9c) is the signal component, and (9d) is the noise component.

We are going to find the optimum transmitter and receiver waveforms

Proof: By definition, the inverse discrete-time Fourier transform of (1) is given by

$$m[k] = \int_{-\frac{1}{2}}^{\frac{1}{2}} M(f) e^{j2\pi fk}\,df.$$

Since $m[0] = \mathsf{E}\{|b[k]|^2\}$, we obtain

$$\mathsf{E}\{|b[k]|^2\} = \int_{-\frac{1}{2}}^{\frac{1}{2}} M(f)\,df.$$

Now, a simple change of variable leads to the conclusion. □

It is worth noting that the power spectral density $M(f)$ is periodic with period 1 because it is the Fourier transform of a discrete-time signal $m[k]$. Consequently, $M(fT) = M((f - l/T)T), \forall l \in \mathbb{Z}$. We will use this property in the next two subsections.

B. Cross Term (9b)

Before we derive the expression for the cross term, we first define the excess bandwidth and the vectorized Fourier transform. The excess bandwidth is a parameter that is used to quantify the extra bandwidth, over the Nyquist bandwidth, spent in pulse shaping. It will be seen that the excess bandwidth determines the degrees of freedom in our optimal waveform design. The vectorized Fourier transform is a representation as a vector-valued function of the ordinary Fourier transform that is a scalar-valued function. This alternate expression of the Fourier transform facilitates our optimal system design in the frequency domain.

Definition 4: The excess bandwidth β is defined by the relation

$$BT = \frac{1+\beta}{2} \tag{12}$$

where B and $1/T$ are, respectively, the bandwidth and the transmission rate of the symbol waveform $s(t)$.

Definition 5: The vectorized Fourier transform $\boldsymbol{x}(f)$ with sampling period $1/T$ of the time function $x(t)$ having bandwidth B is defined as

$$\Big[\; X\left(-\tfrac{L}{T} + f\right) \;\Big]$$

"Calculus of Variations"

Calculus

$\min_{\underline{x}} \; \boxed{f_0(\underline{x})}$

s.t. $\underline{x} \in \Omega$

where $\Re$ denotes the real part, (9a) is the reference component, (9b) is the cross-correlation component between the reference signal and the output, (9c) is the signal component, and (9d) is the noise component.

We are going to find the optimum transmitter and receiver waveforms $s_{\text{opt}}(t)$ and $w_{\text{opt}}(t)$ that jointly minimize the MSE defined as above. Similar to [1], the MSE can be made arbitrarily small by spending more and more power in transmitting the signal. So, it is reasonable to impose a constraint on the average power of the transmitted signal. Hence, the problem is formally stated as

$$\begin{aligned} &\underset{s(t),w(t)}{\text{minimize}} \quad \varepsilon \\ &\text{subject to} \quad \mathbb{E}\left\{\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|\sum_{k=-\infty}^{\infty} b[k]s(t-kT)\right|^2 dt\right\} = A \end{aligned} \quad (10)$$

where $A > 0$ is the average transmitted power.

This problem is inherently a variational problem, where the MSE is a complicated objective functional of two time functions $s(t)$ and $w(t)$. In the next section, we convert this time-domain expression to a frequency-domain expression term by term, to solve the problem in the frequency domain.

III. Frequency-Domain Representation of MSE

A. Reference Component (9a)

Proposition 1: The reference component (9a) in the MSE can be rewritten as

$$\mathbb{E}\{|b[0]|^2\} = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} TM(fT)df. \quad (11)$$

Definition 5: The vectorized Fourier transform $\boldsymbol{x}(f)$ with sampling period $1/T$ of the time function $x(t)$ having bandwidth B is defined as

FEM

$$\boldsymbol{x}(f) \triangleq \begin{bmatrix} X\left(-\frac{L}{T}+f\right) \\ X\left(-\frac{L-1}{T}+f\right) \\ \vdots \\ X\left(\frac{L}{T}+f\right) \end{bmatrix} \quad (13)$$

for $f \in [-1/(2T), 1/(2T))$ with $L \triangleq \lceil \beta/2 \rceil$, where $X(\xi)$ is the Fourier transform[5] of $x(t)$ and β is given by the relation (12).

As shown in Fig. 2, the vectorized Fourier transform requires the sampling of an ordinary Fourier transform in the frequency domain. Note that the sampling rate is T (samples/Hz) and that the argument f serves as the offset of the uniform samples from the integer multiples of $1/T$. In the vectorization, the excess bandwidth determines the number of samples to be taken. As will be later seen, the dimension $2L+1$ is an upper bound on the degrees of freedom in the waveform design.

Proposition 2: The cross term (9b) in the MSE can be rewritten as

$$-2\Re(\mathbb{E}\{b[k]^* z[k]\}) = \int_{-\frac{1}{2T}}^{\frac{1}{2T}} -2\Re\Big(\boldsymbol{w}(f)^H(M(fT)\boldsymbol{p}(f))\Big)df \quad (14)$$

where $\boldsymbol{w}(f)$ and $\boldsymbol{p}(f)$ are, respectively, the vectorized Fourier transform of the receiver waveform $w(t)$ and that of the channel response $p(t)$ of the transmitter waveform.

[5] Throughout this correspondence, a capitalized frequency function represents the Fourier transform of the corresponding time function.

s.t. $x \in \Omega$

$\min_{s(t),w(t)} \ \varepsilon(s(t), w(t))$

s.t. ...

$\min_{f(x)}$ elapsed time

s.t. $\begin{cases} f(0) = h \\ f(\ell) = 0 \\ f \in C^1 \end{cases}$